Forecasting Performance of Alternative Error Correction Models

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Abstract

It is well established that regression analysis on non-stationary time series data may yield spurious results. An earlier response to this problem was to run regression with first difference of variables. But this transformation destroys any long-run information embodied in the levels of variables. According to 'Granger Representation Theorem' (Engle and Granger, 1987) if the variables are co-integrated, there exist an error correction mechanism which incorporates long run information in modeling changes in variables. This mechanism employs an additional lag of the disequilibrium error as an additional variable in modeling changes in variables. It has been argued that ECM performs better than a simple first difference or level regression for long run forecast. This process contributes to the literature in two important ways. Firstly empirical evidence does not exist on the relative merits of ECM arrived at using alternative co-integration techniques. The three popular co-integration procedures considered are the Engle-Granger (1987) two step procedure, the Johansen (1988) multivariate system based technique and the recently developed Autoregressive Distributed Lag based technique of Pesaran et al. (1996, 2001). Secondly, earlier studies on the forecasting performance of the ECM employed macroeconomic data on developed economies i.e. the US, the UK and the G-7 countries. By employing data form the Asian countries and using absolute version of the purchasing power parity and money demand function this paper compares forecast accuracy of the three alternative error correction models in forecasting the nominal exchange rate and monetary aggregate (M2).

1.Introduction

Having reliable forecasts of macroeconomic variables is key information for forming sound macroeconomic growth oriented policies useful for governments, planning and development agencies, central banks, long term direct and portfolio investors and other relevant stakeholders. This paper demonstrates which error correction techniques yield better forecasts of the macro variables. Forecasts are based on past information contained in past trend of the historical data. If forecasts are too far away from the historical trends, they are indicative of important information regarding some events which have altered the historical path of the economy.

The Granger Representation Theorem (Engle and Granger, 1987) enables simultaneous modeling of first difference and the level of the variables using an error correction mechanism which provides the framework for estimation, forecasting and testing of co-integrated systems. If X_t and Y_t are co-integrated and individually I(1) variables with co-integration vector $(1,-\beta_0,-\beta_1)$ the general form of the ECM can be expressed as

$$A(L)\Delta Y_t = \delta + B(L)\Delta X_t + \alpha(Y_{t-1} - \beta_0 - \beta_1 X_{t-1}) + u_t \qquad (1)$$

with the lag polynomials

$$A(L) = 1 - a_1 L - a_2 L^2 - \ldots - a_p L^p \,;\, B(L) = b_0 + b_1 L + b_2 L^2 + \ldots + b_q L^q$$

where the lag operator is defined as $L^i Y_t = Y_{t-i}$. In this model the coefficients in the *A(L)* and *B(L)* represent the impact of short run changes while the long run effects are given by the co-integration vector $(1,-\beta_0,-\beta_1)$ and the α controls the speed of adjustment of short run changes towards long run path.

As co-integration and ECM provides a unified framework of modeling both long and short run an interesting question for researcher was whether incorporating the long-run restriction in an error correction model yields superior forecast in comparison with pure first difference model which do not impose co-integration restriction. On a theoretical ground co-integration is expected to yield better forecast as pointed by Stock (1995, p-1) who asserts that “If variables are co-integrated, their values are linked over the long run, and imposing this information can produce substantial improvement in forecast over long horizons”. This assertion is based on theoretical results by Engle and Yoo (1986) that long horizon forecasts from the co-integrated systems satisfy the co-integration relationship exactly and that the cointegration combination of variables can be forecast with finite long-horizon forecast error variance.

A simulation study by Engle and Yoo (1987) shows that the two step EG ECM provide better forecast compared to unrestricted VAR particularly at longer horizons while a similar simulation study by Chambers (1993) further corroborated this result using a non-linear one-step ECM. Using the same experimental set up as in Engle and Yoo, Clements and Hendry (1995) find that over-differencing the system results in inferior forecasting performance. In a simulation study using a four-dimensional VAR(2) Reinsel and Ahn (1992) show that forecast gains from co-integrated system depends on proper specification of the number of unit roots and under specifying the number of unit roots results in poor performance for ten to twenty five steps ahead forecasts whereas over-specification results in inferior short-term forecasts.

After the pioneering two-step estimator of the ECM parameters proposed by Engle and Granger (1987) several ECM techniques have been developed. The Engle-Granger technique can identify only a single equilibrium relationship among the variables under study. Johansen (1988) proposed a framework of estimation and testing of vector error correction model (VECM) based on vector auto regression (VAR) equations. The VECM can be expressed as:

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{j=1}^{p} \Gamma_j Y_{t-j} + u_t \tag{2}$$

The Π is a $g \times g$ matrix containing the long-run parameters. If there are r co-integration vectors then Π can be expressed as a product of two matrices as $\Pi = \alpha\beta'$ where both α and β are $g \times r$ matrices. The matrix β contains the coefficients of long-run relationship and α contains the speed of adjustment parameters which are also interpreted as the weight with which each co-integration vector appears in a given equation. This approach can accommodate multiple equilibrium relationships in the VECM.

Both of these estimation techniques assume that the variables to be modeled are I(1). Recently Pesaran, Shin and Smith (1996) and Pesaran (2001) proposed a technique based on Autoregressive Distributed Lag (ARDL) model which allows both I(0) and I(1) variables thus potentially avoids pre-test bias.

To explain the three main techniques of error correction model we consider the real money balance relationship. The long run relationship is expressed as:

$$MP_t = \beta_0 + \beta_1 y_t + \beta_2 i_t + u_t \tag{3}$$

where $MP = log(M2/CPI)$, $y = log(output)$, i =*nominal interest rate*

The Engle Granger technique uses residuals $EC_t = \hat{u}_t = MP_t - \hat{\beta}_0 - \hat{\beta}_1 y_t - \hat{\beta}_2 i_t$ from the long run equation and test for stationarity of the residuals. Co-integration exists if EC_t is stationary. The error correction model will then be formulated as:

$$\Delta MP_t = \gamma_0 + \sum_{i=1}^{m_1} \gamma_{1i} \Delta MP_{t-i} + \sum_{i=1}^{m_2} \gamma_{2i} \Delta y_{t-i} + \sum_{i=1}^{m_3} \gamma_{3i} \Delta i_{t-i} + \alpha\, EC_{t-1} + v_t \qquad (4)$$

The Johansen's (1988) technique employs the Vector Error Correction Model (VECM)

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-1} + \omega_t \qquad (5)$$

Where Π and Γ_i are square matrices whose elements depend on the coefficients of long run model and Y_t contains the endogenous variables of the model. In present case of money balance equation $Y_t = [MP_t\ y_t\ i_t]'$. A test of rank of Π then establishes the number of co-integration relationships to enter in the VECM equation. If there are '*r*' co-integration relationships, the matrix Π is expressed as product of two matrices each of which is of order $g \times r$ i.e. $\Pi = \alpha\beta'$. For example if r = 1, the VECM will be written as (for g = 3 variable system)

$$\Delta Y_t = \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \alpha_{13} \end{bmatrix} (\beta_{11} y_{1t-1} + \beta_{12} y_{2t-1} + \beta_{13} y_{3t-1}) + \sum_{i=1}^{k} \Gamma_i \Delta y_{t-1} + \omega_t \qquad (6)$$

Where $y_1 = MP$ and $y_2 = y$ and $y_3 = i$

For testing co-integration the ARDL technique specifies the dynamic equation as

$$\Delta MP_t = \gamma_0 + \sum_{i=1}^{m_1} \gamma_{1i} \Delta MP_{t-i} + \sum_{i=1}^{m_2} \gamma_{2i} \Delta y_{t-i} + \sum_{i=1}^{m_3} \gamma_{31} \Delta r_{t-i} + \alpha\, MP_{t-1} + \beta\, y_{t-1} + \delta\, i_{t-1} + \eta_t \qquad (7)$$

If there is no co-integration, $\alpha = \beta = \delta = 0$. The corresponding F-test has non-standard asymptotic distribution. Pesaran et al. (1996) provide two sets of asymptotic critical values for the test. One set assumes that all variables are I(0) and the other assumes they are all I(1) variables. If the computed F-statistic falls above the upper bound critical value, then the null of no co-integration is rejected. If it falls below the lower bound, then the null cannot be rejected. Finally, if it falls inside the critical value band, the result would be inconclusive. These two sets of critical values refer to two polar cases but actually provide a band covering all possible classifications of the variables into I(0), I(1) or even fractionally integrated variables. Once co-integration is established in the second stage the error correction model is formulated as:

$$\Delta MP_t = \delta_0 + \sum_{i=1}^{m_1} \delta_{1i} \Delta MP_{t-i} + \sum_{i=1}^{m_2} \delta_{2i} \Delta y_{t-i} + \sum_{i=1}^{m_3} \delta_{3i} \Delta r_{t-i} + \phi EC_{t-1} + \eta_t \tag{8}$$

Where error correction term *EC* is formulated by normalizing the long run coefficients in (7). In all these cases the optimal lags m_1, m_2 and m_3 may be selected by employing information criteria.

In the literature some studies have compared forecast ability of the error correction models resulting from the Engle-Granger and the Johansen VECM technique. However the literature does not provide empirical evidence regarding the forecast accuracy of the ARDL based error correction model and its comparison with EG and Johansen techniques. In addition, most of the empirical evidence employing real data in forecast comparison comes from the developed economies. This study provides empirical evidence of forecasting performance of the alternative error correction models resulting from the three techniques using the data from Asian countries.

2. The Literature

Hoffman and Rasche (1996) compared the forecasting performance of a co-integrated system relative to the forecasting performance of a comparable VAR that fails to recognize that the system is characterized by co-integration. They considered co-integrated system composing three vectors, a money demand representation, a Fisher equation, and a risk premium captured by an interest rate differential. The data were from the US economy. They found that the advantage of imposing co-integration appears only at longer forecast horizon and this is also sensitive to the appropriate data transformation. They considered eight years out-sample forecast horizon.

Jansen and Wang (2006) investigated the forecasting performance of the error correction model arising from the co-integration relationship between the equity yield on the S&P 500 index and the bond yield relative to that of univariate models. They found that the Fed Model improves on the univariate model for longer-horizon forecasts, and the nonlinear vector error correction model performs even better than its linear version. They considered ten years forecast horizon.

Wang and Bessler (2004) employed five agriculture time series from the US. They used annual data from 1867 to 1966 for model specification and the data for 1966 to 2000 were used for out-of sample forecast evaluation. Their results favored ECM for three to four year ahead forecast. However the differences in forecast obtained from various models were not statistically significant.

Lin and Tsay (1996) considered both simulated data and financial and macroeconomic real data from the UK, Canada, Germany, France and Japan and interest rate data from the US and Taiwan. Their results are contradictory as the simulated data yield better forecast from the ECM whereas the performance of ECM for real data is mixed. They attribute this contradiction to deficiency in forecast error measure which does not recognize that forecast are tied together in the long-run.

This brief literature review indicates that at best the results on relative merit of imposing co-integration constraint are mixed. If there is some advantage of using the ECM it occurs at longer horizon only. This review also indicates that very few studies employ data from the less developed economies such as the East and South Asian economies. Also no study has yet considered forecasting performance of the newly developed ARDL based co-integration. It has been argued (e.g. Narayan and Narayan, 2005) that ARDL has important advantages over the Engle-Granger and Johansen approaches. Firstly, it can be applied regardless of whether underlying variables are I(0) or I(1). Secondly, ARDL approach has better small sample properties than the EG and Johansen co-integration tests. Thirdly appropriate modification of the orders of the ARDL model is sufficient to simultaneously correct for residual serial correlation and the problem of endogenous variables.

3.The data and the models

The economic models we considered are the Purchasing Power Parity (PPP) and the demand for real money balances function. The absolute PPP states that exchange rate between two currencies adjust to remove any arbitrage opportunities (buy in a low price market and sell with a profit in a high price market). If PPP holds then in the long run exchange rate equals the ratio of price level in the two economies. i.e. the intercept equals zero and slope equals 1 in the equation:

$$\log(e)_t = \beta_0 + \beta_1 \log(CPI\ Ratio)_t + u_t \tag{9}$$

Secondly we considered demand of real money balances '*M/CPI*' depends positively on transaction volume i.e. output level '*Y*' and negatively on cost of holding cash i.e. nominal interest rate '*i*' i.e.

$$\log(M_t / CPI_t) = \beta_0 + \beta_1 \log(Y_t) + \beta_2 i_t + u_t \tag{10}$$

Thus the task is to forecast exchange rate (local currency per dollar) and money stock (M2) from the alternative ECM resulting from the three co-integration techniques. The quarterly data (1978Q1-2009Q4) of ten Asian countries are employed namely

1. Korea 2. Singapore 3. Malaysia 4. Indonesia 5. Thailand 6. Philippines 7. Sri Lanka 8. India 9. Pakistan 10. Bangladesh.

Interest rate is measured by discount rate, lending rate or money market rate (whichever is available for full sample period). Output is measured by manufacturing production index which indicate significant seasonality in some countries so quarterly dummies are added in estimation. The data comes mostly from International Financial Statistics (IFS). Thai manufacturing production index is obtained from Bank of Thailand. The output data for Sri Lanka are not available so money demand results are not presented for Sri Lanka.

The empirical analysis possess certain challenges e.g. EG and Johansen require the pre-testing for unit root in the variables and strictly speaking are valid if variables are I(1).

However ARDL does not need such pre-testing. Unit root tests on all the series were conducted using ADF, Phillips-Perron and KPSS methods. In some cases the EG and Johansen's co-integration is not strictly applicable since the order of integration was not the same for the variables under study. However the lack of power of unit roots tests is well known especially as the autoregressive coefficient approaches unity. We therefore proceed to co-integration analysis in these cases as well[1]. In some cases EG, Johansen and ARDL co-integration tests could not uncover any co-integration. In these cases we followed the recommendation of Kremers et al. (1992) and Bannerjee et al. (1998) who have argued that a significant lagged error term is a relatively more efficient way of establishing co-integration. In most of the cases in our study the co-integration evidence comes from significance of Error Correction term. The analysis is conducted for all countries despite these limitations.

We considered quarterly data from 1978Q1 to 2009Q4. For model specification and estimation we employ data from 1978Q1 to 1994Q4 and the forecast evaluation is conducted for the period 2005Q1 and 2009Q4. We employ Mean Absolute Percentage Error (MAPE) to evaluate the forecast accuracy. This measure eliminates the effect of scaling of variables so that forecast error from countries is comparable. The MAPE is given by:

$$MAPE = 100 \times \frac{1}{H} \sum_{t=1}^{H} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \tag{11}$$

[1] Some studies on co-integration assume that all series are I(1) despite some evidence against this pre-requisite condition. For example MacDonald and Taylor (1991) for the case of Germany, Japan, and the UK, found the evidence that some series may be stationary around a trend. They proceed for co-integration to avoid the conflicting inference of degree of integration of all variables not being the same.

Where Y_t and $\hat{Y}_t$ represent actual and forecast respectively and H represent forecast horizon.

4. Results and Discussion

The following tables (Table 1 and Table 2) present the comparison of forecast accuracy based on MAPE. The best ECM model in each case is highlighted. Generally the ARDL appears to yield lower forecast errors followed by Johansen technique. This is the case for money stock forecast (Table 2) where potentially more than one co-integration vectors are possible. For Bangladesh the EG ECM yields the best forecast for the two variables. For Malaysia Johansen technique appears to be superior. For India and Singapore the ARDL technique results in the lowest forecast error. The results for other countries are mixed for the two variables.

Table 1: MAPE of exchange rate forecast for five year forecast horizon

COUNTRIES	Engle Granger	Johansen	ARDL
Bangladesh	**3.058**	**3.552**	**5.887**
India	**14.063**	**15.265**	**5.187**
Indonesia	**62.365**	**40.818**	**13.938**
Korea	**17.628**	**13.969**	**14.598**
Malaysia	**33.750**	**15.943**	**33.735**
Pakistan	**7.325**	**9.600**	**6.697**
Philippines	**37.53**	**41.039**	**27.723**
Singapore	**8.69**	**12.195**	**8.427**
Sri Lanka	**20.563**	**17.023**	**17.366**
Thailand	**16.730**	**14.752**	**16.227**
Average	**22.5%**	**18.4%**	**15%**

Notes: Schwarz criteria selects lag 1 as optimal for Engle-Granger method for all ten countries. Regression of ECM model with this optimal lags indicate that error correction term is insignificant only in Sri Lanka. For VECM estimation using Johansen technique optimal lags are obtained by choosing lags based on AIC criteria and then determined using AIC then insignificant lags were removed using joint F-test. Same number of lags for each variable was employed in this case. Trace and Max tests did not provide evidence of cointegration in some cases but subsequent analysis by VECM models indicate that loading coefficients was insignificant only in Indonesia. In other cases loading coefficient was significant with negative sign in at least one VECM equation. Optimal lags using Schwarz criteria for ARDL is 1 for all countries. With optimal lags ECT term is insignificant only in Indonesia.

Table 2: MAPE of M2 forecast for five year forecast horizon

COUNTRIES	Engle Granger	Johansen	ARDL
Bangladesh	**4.178**	**12.584**	**5.835**
India	**14.532**	**11.425**	**8.047**
Indonesia	**10.601**	**9.334**	**11.839**
Korea	**22.725**	**19.906**	**10.982**
Malaysia	**8.374**	**5.811**	**6.747**
Pakistan	**5.074**	**7.560**	**6.321**
Philippines	**19.403**	**4.951**	**7.629**
Singapore	**2.204**	**2.225**	**2.056**
Thailand	**35.399**	**15.1536**	**6.166**
Average	**13.6%**	**9.9%**	**7.3%**

Notes: Optimal lags for Engle-Granger test are 1 for all counties. In some countries Engle-Granger ADF test did not uncover co-integration but subsequent in ECM model error correction term is insignificant only in Korea, Malaysia and Pakistan.
Optimal lags for Johansen vary over different countries using same lags for each variable. Trace and Max statistics do not indicate co-integration but in VECM models the loading coefficients was insignificant only in Indonesia.Optimal lags using Schwarz criteria using ARDL method are four for Korea, Philippines, Pakistan and Bangladesh; three for Singapore and one for India, Malaysia, Thailand. With optimal lags error correction term is insignificant only in Malaysia and Pakistan
Manufacturing production for Sri Lanka is not available so money demand estimation is not possible.

5. Conclusion

It is well known that regression analysis on non-stationary time series data may be spurious (non-sense) if the underlying variables are not co-integrated. Error correction models provide a convenient framework for estimation, testing and forecasting. However various co-integration estimation and testing techniques have been developed in the literature. In this paper we have compared the forecasting accuracy of three popular error correction models that are derived from the Engle-Granger, Johansen and the ARDL techniques. The results indicate that in general the ECM based on both the ARDL and Johansen techniques outperform the Engle-Granger technique. The ARDL ECM results in the best performance in about 48% of the cases whereas the Johansen's ECM yields the best performance in about 36% cases. The ARDL technique appears to be superior even in cases where more than one co-integration relationships are possible i.e. money demand model which involve three variables in the system. The average MAPE for exchange rate forecast across ten countries is 15%, 18.4% and 22.5% for the ARDL, Johansen and the EG techniques respectively. The average MAPE for M2 forecasts are 7.3%, 9.9% and 13.6% for the ARDL, Johansen and the EG techniques respectively. Thus our analysis provides comparatively better evidence in favor of the ARDL based ECM. Also it will be interesting to compare forecast of ECM from alternative forecasting techniques which do not impose co-integration e.g. ARIMA and VAR techniques. Moreover investigating accuracy of short run forecast of up to four quarters may be investigated. Current research is progressing in these directions.

References

Bannerjee, A., Dolado and R. Mestre (1998), "Error-correction mechanism tests for cointegration in single equation framework", Journal of Time Series Analysis, 19, 267-283.

Chambers, M.J. (1993), "A note on forecasting in co-integrated systems". Computers Methods and Applications", 25, 93-99.

Clements, M. P. and D. F. Hendry (1995), "Forecasting in co-integrated systems". Journal of Applied Econometrics10, 127-146.

Engle, R.F. and Granger, C.W.J. (1987),"Co-integration and error correction: Representation, estimation and testing". *Econometrica* 55,251-276

Engle, R.F. and Yoo, B.S. (1987), "Forecasting and testing in co-integrated systems". *Journal of Econometrics,* 35,143-159.

Haigh, M.S. (2000), "Co-integration, unbiased expectations and forecasting in the BIFFEX freight futures market". Journal of Futures Market, 20, 545-571

Hoffman, D.L and Rasche, R. H. (1996), "Assessing Forecast Performance in a Cointegrated System". Journal of Applied Econometrics, Vol. 11,495-517.

Jansen, D.W and Wang, Z. (2006), "Evaluating the 'Fed Model' of Stock Price Valuation: an Out-of-Sample Forecasting Perspective". Econometric Analysis of Financial and Economic Time Series/Part B Advances in Econometrics, Volume 20, 179–204.

Johansen, S. (1998). "Statistical analysis of cointegrating vectors". Journal of Economics of Economics Dynamics and Control. 12, 231-54.

Kremers, J., Ericsson, N.R. and Dolado, J. (1992), "The power of Cointegration tests", Oxford Bulletin of Economics and Statistics, 54: 325-343.

Lin, J.L. and Tsay, R.S. (1996), "Co-integration constraint and Forecasting: An empirical examination". Journal of Applied Econometrics, Vol. 11,519-538

MacDonald, R., and Taylor, M.P. (1991), "The Monetary Approach to the Exchange Rate", Economic Letters 37, 179-85.

Narayan, P.K and Narayan, S. (2005), "Estimating income and price elasticities of imports for Fiji in a cointegration framework", Economic Modelling, 22, 423-438.

Pesaran, M. H., Shin, Y., and Smith, R. J. (1996), "Bounds Testing Approaches to the Analysis of Level Relationships" DEA working paper 9622, Department of Applied Economics, University of Cambridge.

Pesaran, M. H., Shin, Y. and Smith, R. J. (2001), "Bounds Testing Approaches to the Analysis of Level Relationships". Journal of Applied Econometrics 16, 289-326.

Stock, J.H., (1995), "Point forecasts and prediction intervals for long-horizon forecast". Manuscript, J.F.K. School of Government, Harvard University.

Wang, Z and Bessler, D.A. (2004)," Forecasting performance of multivariate time series models with full and reduced rank: An empirical examination". International Journal of Forecasting 20, 683-695