Methods for Solving Nonhomogeneous Linear Differential Equations with Constant Coefficients

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Abstract:

The general solution of a second-order nonhomogeneous differential equation with constant coefficients is normally structured as a sum of the complementary solution of the homogeneous equation and a particular solution of the nonhomogeneous equation. There are three reliable methods which are commonly used for solving nonhomogeneous linear equations. The first of these methods, **reduction of order**, can be used to solve any second-order linear equation provided that one nontrivial solution in the complementary solution part is somehow known. The second method, **variation of parameters**, requires knowing two independent solutions in the complementary solution part. The third method, **the method of undetermined coefficients**, applies to a nonhomogeneous linear differential equation with constant coefficients. Unlike reduction of order and variation of parameters, this method applies to a restricted class of equations.

A somewhat not well known method, **reduction to first order method (RFO)**, requires a suitable transformation to reduce the order of the second-order linear nonhomogeneous differential equation with constant coefficients. This reduced order differential equation can then be solved routinely using the integrating factor technique. We strongly advocate the teaching of **reduction to first order method** in the classroom setting. To illustrate the general utility of the method several examples are presented. This method is also applicable to linear equations of higher order with constant coefficients.