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Holographic dark energy models in higher derivative torsion corrected modified teleparallel gravity

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We consider the recently proposed higher derivative torsion corrected modified teleparallel gravity and holographic dark energy (HDE) models. We apply the correspondence scheme to construct models in underlying scenario using various scale factor forms. We investigate the reconstructed functions through equation of state (EoS) parameter. It is demonstrated that the EoS parameter provides quintom-like nature of the Universe in most of the cases, i.e. it drives the Universe from vacuum dark energy era toward phantom era of the Universe by crossing the phantom divide line. We also demonstrate that the consistency with the observational data can be achieved.

Keywords: Higher order torsion corrected modified teleparallel gravity; holographic dark energy models; cosmological parameters.

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1. Introduction

One of the perplexing issues in modern cosmology is that our Universe is undergoing the accelerated expansion which is due to some kind of exotic force, called dark energy (DE) which pulls apart matter formations at a galactic scale. Various observational schemes (such as type Ia Supernovae, large scale structure, WMAP, etc.) reveal these properties of DE as well as proposed some specific values of equation of state (EoS) parameter [1–10]. This parameter relates the energy density with pressure and its negative value corresponds to DE era of the Universe. The nature of this force is still under observations. The search for best fit source of DE has become an active field of modern cosmology.

In order to describe the accelerated expansion phenomenon, two different approaches have been adopted. One is the proposal of various dynamical DE models

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such as family of Chaplygin gas [11], holographic [12, 13], new agegraphic [14], pilgrim [15] DE models, etc. The holographic dark energy (HDE) is one of the most interesting dynamical model and is based on the holographic principle proposed in [16]. This model has been constrained and tested by various astronomical schemes and in conjunction with anthropic principle [12, 13]. By the inclusion of holographic principle into cosmology, it can be found the upper bound of the entropy contained in the Universe [13, 17]. Through this bound, Li [13] proposed the constraint on the DE density:

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2}, \tag{1}$$

where c, L, $M_p = (8\pi G)^{-1/2} = 10^{18}\,\mathrm{GeV}$ indicate the numerical constant, IR cutoff, reduced Planck mass, respectively. By choosing the Hubble horizon as an IR cutoff, then the energy density of HDE turns out to be

$$\rho_{\mathrm{DE}} = 3c^2 M_P^2 H^2, \tag{2}$$

where we assume $M_p = 1$ in further calculations.

A second approach for understanding this strange component of the Universe is modifying the standard theories of gravity, namely, general relativity (GR) or teleparallel theory equivalent to general relativity (TEGR). Several modified theories of gravity are $f(R)$, $f(T)$ [18–26], $f(R,\mathcal{T})$ [27, 28], $f(G)$ [29–36] (where R is the curvature scalar, T denotes the torsion scalar, $\mathcal{T}$ is the trace of the energy–momentum tensor and G is the invariant of Gauss–Bonnet defined as $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$). For clear review of DE models and modified theories of gravity, see [8–10].

The proposals of modified gravity have been started from the standard curvature-based gravitational formulation and modify the Einstein–Hilbert action with $F(R)$ (this is the simplest extended model) [8–10]. In the similar way, $F(T)$ has been built starting from TEGR formulation. It is noted that the structure of TEGR (at the level of equations) is completely equivalent with general relativity, $F(T)$ is a different class of modified gravity than $F(R)$ gravity, and therefore its cosmological implications bring novel features, either at late times [18–26] or at the inflationary epoch [37, 38]. Inspired by the corresponding curvature-based modification [39, 40], Otalora and Saridakis [41] constructed the novel torsional gravitational modifications using higher derivative, $(\nabla T)^2$ and $\Box T$ terms, i.e. theories that are characterized by the Lagrangian $F(T,(\nabla T)^2,\Box T)$. In this work, we consider $F(T,(\nabla T)^2,\Box T)$ gravity and reconstructed the models by assuming three forms of HDE models and four forms of scale factors. We investigate these models through EoS parameter.

The outline of the paper is the following: In next section, we elaborate the underlined gravity, HDE models and four forms of scale factor. In Sec. 3, the reconstruction scenario will be developed. In the last section, we summarized our results.

2. $F(T, (\nabla T)^2, \Box T)$ Gravity and Dark Energy Models

In this gravity, the Friedmann equations can be described in the standard form as follows:

$$3H^2 = \rho_{\mathrm{DE}} + \rho_m, \quad -2\dot{H} = \rho_{\mathrm{DE}} + \rho_m + p_{\mathrm{DE}} + p_m, \tag{3}$$

where ρ_{DE} and ρ_m are the energy densities of DE and dark matter (DM) while p_{DE} and p_m are the pressures of DE and DM, respectively. Also, the energy density and pressure of DE density can be written as

$$\begin{aligned}\rho_{\mathrm{DE}} = {} & -\frac{F}{2} - 6H^2F_T + 3H^2 - 6F_{X_2}\dot{H}^2 - 6H(F_{X_2} + 24H^2F_{X_1})(3H\dot{H} + \ddot{H}) \\ & - 144H^3\dot{H}\dot{F}_{X_1} - 6H(3H^2 - \dot{H})\dot{F}_{X_2} - 6H^2\ddot{F}_{X_2},\end{aligned} \tag{4}$$

$$p_{\mathrm{DE}} = (\gamma - 1)\rho_{\mathrm{DE}} - 3\gamma H^2 - 2\dot{H}. \tag{5}$$

In the second equation, EoS $p_m = (\gamma - 1)\rho_m$ ($\gamma = \text{constant}$) is being utilized. Also, $T = -6H^2$, $X_1 = 144H^2\dot{H}$ and $X_2 = -12(\dot{H}(\dot{H} + 3H^2) + H\ddot{H})$.

2.1. *Dark energy models*

To realize the role of DE in modified gravity, several cosmological scenarios have been built by utilizing the useful technique as proposed by [42–45]. There exists several DE EoSs in the literature, however, author [41] has discussed the more general forms of DE inhomogeneous EoS. In view of these EoS, they have usefully remarked that the more general form may contain the derivative of H, like $\dot{H}$, $\ddot{H}, \ldots$ in principle, it is given by

$$F(p,\ \rho,\ H,\ \dot{H},\ \ddot{H}) = 0.$$

This form contains a family of Chaplygin gas and much more complicated EoS. They have also investigated its non-trivial scenario and sketched a useful picture of cosmological implications. They pointed out that the inhomogeneous term in EoS helps to realize the crossing of phantom barrier. In the present scenario, we will consider three forms of HDE models which are described below.

2.2. *Holographic DE with Hubble horizon*

The Hubble horizon is the first IR cutoff which remains under criticism due to its inconsistent behavior with current cosmic acceleration [13]. The deficiency with this model is settled down by proposing that HDE with this IR cutoff has ability to represent the cosmic acceleration by taking interaction scenario between DE and DM [46, 47]. This HDE model with Hubble horizon is also tested through various observational schemes [48, 49]. Also, Sheykhi [50] has investigated this model by taking interaction with CDM and pointed out that such model possesses the ability to explain the present scenario of the Universe.

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2.3. *HDE with event horizon*

This model was proposed by Li [13] and suggested that HDE with event horizon provides the consistent results with observational data with the help of HDE parameter. Later on, many discussions about cosmic acceleration have been made by choosing this HDE model which provide different constraints on EoS parameter [17, 51, 52]. The validity of thermodynamics laws has also been discussed by taking event horizon as a boundary of cosmological system [52–56]. In addition, different cosmological schemes have been used to check the viability of HDE with event horizon [57–61].

The HDE density with event horizon can be defined as follows [62]:

$$\rho_{\mathrm{DE}} = \frac{3c^2}{R_h^2}, \tag{6}$$

where R_h represents the future event horizon which is defined as

$$R_h = a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2}. \tag{7}$$

The derivative of R_h with respect to t is given by

$$\dot{R}_h = R_h H - 1. \tag{8}$$

2.4. *New holographic dark energy*

We consider new holographic dark energy (NHDE) model in terms of Hubble parameter and its derivative form. This model is referred as NHDE model with energy density as [42]

$$\rho_{\mathrm{DE}} = 3(\eta H^2 + \xi\dot{H}). \tag{9}$$

Here, η and ξ are constant parameters. This type of IR cutoff is motivated from the Ricci scalar of the FRW Universe in order to avoid causality problem. This IR cutoff has been widely utilized in the literature in order to discuss the cosmic acceleration in GR as well as modified theories of gravity.

2.5. *Cosmic scale factors*

Here, we will give brief description of some well-known factors for elaborating our cosmological study.

2.5.1. *Power law scale factor*

We consider the scale factor in the form

$$a(t) = a_0 t^m, \tag{10}$$

where $m > 0$. Subsequently, Hubble parameter H and its time derivative $\dot{H}$ are

$$H = \frac{m}{t}, \quad \dot{H} = -\frac{m}{t^2}. \tag{11}$$

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The EoS parameter corresponding to this scale factor can be obtained by using following relation:

$$\omega_{\rm DE} = \frac{p_{\rm DE}}{\rho_{\rm DE}}. \tag{12}$$

By using the relations of scale factor, (4) and (5), we can obtain EoS parameter as

$$\begin{aligned}
\omega_{\rm DE} = \Bigg(&\frac{2m}{t^2} - \frac{3m^2}{t^2} + (-1+\gamma)\Bigg(\frac{3m^2}{t^2} + t^{-3m} - \frac{F[t]}{2} - \frac{1}{2}tF'[t] + \frac{tF'[t]}{24(-1+m)} \\
&- \frac{6m}{t}\left(\frac{2m}{t^3} - \frac{3m^2}{t^3}\right)\left(-\frac{t^5F'[t]}{36m^2} - \frac{t^5F'[t]}{144(-1+m)m^2}\right) - \frac{6m}{t}\left(\frac{m}{t^2} + \frac{3m^2}{t^2}\right) \\
&\times \left(-\frac{5t^4F'[t]}{144(-1+m)m^2} - \frac{t^5F''[t]}{144(-1+m)m^2}\right) + \frac{144m^4}{t^5}\Bigg(-\frac{7t^6F'[t]}{864m^4} \\
&- \frac{t^7F''[t]}{864m^4}\Bigg) - \frac{6m^2}{t^2}\Bigg(-\frac{5t^3F'[t]}{36(-1+m)m^2} - \frac{5t^4F''[t]}{72(-1+m)m^2} \\
&- \frac{t^5F^{(3)}[t]}{144(-1+m)m^2}\Bigg)\Bigg)\Bigg)\Bigg(\frac{3m^2}{t^2} + t^{-3m} - \frac{F[t]}{2} - \frac{1}{2}tF'[t] + \frac{tF'[t]}{24(-1+m)} \\
&- \frac{6m}{t}\left(\frac{2m}{t^3} - \frac{3m^2}{t^3}\right)\left(-\frac{t^5F'[t]}{36m^2} - \frac{t^5F'[t]}{144(-1+m)m^2}\right) - \frac{6m}{t}\left(\frac{m}{t^2} + \frac{3m^2}{t^2}\right) \\
&\times \left(-\frac{5t^4F'[t]}{144(-1+m)m^2} - \frac{t^5F''[t]}{144(-1+m)m^2}\right) \\
&+ \frac{144m^4}{t^5}\left(-\frac{7t^6F'[t]}{864m^4} - \frac{t^7F''[t]}{864m^4}\right) \\
&- \frac{6m^2\left(-\frac{5t^3F'[t]}{36(-1+m)m^2} - \frac{5t^4F''[t]}{72(-1+m)m^2} - \frac{t^5F^{(3)}[t]}{144(-1+m)m^2}\right)}{t^2}\Bigg)^{-1}.
\end{aligned}$$

2.5.2. *Bouncing scale factor*

In the context of bouncing cosmology, the Universe is driven from a contracting epoch ($H < 0$) to an expanding epoch ($H > 0$). This behavior predicts a transitionary inflationary Universe which is a solution for flatness problem in big-bang cosmology. Also, bouncing solutions have been widely discussed in various gravities [63–68]. The bouncing scale factor can be defined as follows [69]:

$$a(t) = a_0 + b_2(t-t_0)^{2n}, \quad H(t) = \frac{2n\alpha(t-t_0)^{2n-1}}{a_0 + \alpha(t-t_0)^{2n}}, \quad n = 1, 2, 3, \ldots, \tag{13}$$

where a_0, α appear as positive (dimensional) constants and n represents the positive natural number. The bouncing time is fixed at $t = t_0$. The scale factor exhibits the decreasing behavior for $t < t_0$ and shows contraction of the Universe with negative

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Hubble parameter. While it shows increasing behavior for $t > t_0$ which implies the expansion of the Universe with positive Hubble parameter.

2.5.3. *Intermediate scale factor*

We choose this scale factor because it shows consistency with astrophysical observations [70–72]. It also plays a key role in the cosmological analysis while a hypothetical scale factor may not be consistent with the inflationary scenario. The intermediate form of scale factor can be defined as follows [70]:

$$a(t) = e^{b_1 t^{\beta}}, \quad 0 < \beta < 1, \tag{14}$$

where b_1 is a constant. The corresponding Hubble parameter is

$$H(t) = b_1 \beta t^{\beta-1}. \tag{15}$$

In this case, EoS parameter leads to

$$\begin{aligned}
\omega_{\mathrm{DE}} = {} & \Bigg(2\Bigg(-2t^{-2+\beta}(-1+\beta)\beta b_1 - 3t^{-2+2\beta}\beta^2 b_1^2 + \frac{1}{2}(-1+\gamma)\Bigg(2e^{-3t^{\beta}b_1} \\
& - F[t] + 6t^{-2+2\beta}\beta^2 b_1^2 + \frac{tF'[t]}{-1+\beta} + \frac{t(-1+\beta)F'[t]}{2(-2+\beta)(-3+2\beta)+3t^{\beta}\beta(-4+3\beta)b_1} \\
& - \frac{t(-2+\beta+3t^{\beta}\beta b_1)(9-9\beta+2\beta^2+3t^{\beta}\beta(-4+3\beta)b_1)F'[t]}{(3-5\beta+2\beta^2)(2(6-7\beta+2\beta^2)+3t^{\beta}\beta(-4+3\beta)b_1)} \\
& - \frac{t((7-4\beta)F'[t]+tF''[t])}{3-5\beta+2\beta^2} + t(1-\beta+3t^{\beta}\beta b_1)(-2(-2+\beta)(-3+2\beta) \\
& \times \frac{((-5+2\beta)F'[t]-tF''[t]) - 3t^{\beta}\beta(-4+3\beta)b_1((-5+3\beta)F'[t]-tF''[t]))}{(-1+\beta)(2(-2+\beta)(-3+2\beta)+3t^{\beta}\beta(-4+3\beta)b_1)^2} \\
& + (t(9t^{2\beta}(4-3\beta)^2\beta^2 b_1^2((-5+3\beta)((-4+3\beta)F'[t]-2tF''[t])+t^2F^{(3)}[t]) \\
& + 4(3-2\beta)^2(-2+\beta)^2(2(-5+2\beta)((-2+\beta)F'[t]-tF''[t])+t^2F^{(3)}[t]) \\
& + 6t^{\beta}(-2+\beta)\beta(-3+2\beta)(-4+3\beta)b_1((40+\beta(-45+11\beta))F'[t]+2t \\
& \times(-5(-2+\beta)F''[t]+tF^{(3)}[t]))))/((-1+\beta)(2(-2+\beta)(-3+2\beta)+3t^{\beta}\beta \\
& \times(-4+3\beta)b_1)^3)\Bigg)\Bigg)\Bigg)\Bigg(2e^{-3t^{\beta}b_1} - F[t] + 6t^{-2+2\beta}\beta^2 b_1^2 + \frac{tF'[t]}{-1+\beta} \\
& + \frac{t(-1+\beta)F'[t]}{2(-2+\beta)(-3+2\beta)+3t^{\beta}\beta(-4+3\beta)b_1} - (t(-2+\beta+3t^{\beta}\beta b_1) \\
& \times(9-9\beta+2\beta^2+3t^{\beta}\beta(-4+3\beta)b_1)F'[t])((3-5\beta+2\beta^2) \\
& \times(2(6-7\beta+2\beta^2)+3t^{\beta}\beta(-4+3\beta)b_1))^{-1} - \frac{t((7-4\beta)F'[t]+tF''[t])}{3-5\beta+2\beta^2} \\
& + (t(1-\beta+3t^{\beta}\beta b_1)(-2(-2+\beta)(-3+2\beta)((-5+2\beta)F'[t]-tF''[t])
\end{aligned}$$

$$
\begin{aligned}
&- 3t^{\beta}\beta(-4+3\beta)b_1((-5+3\beta)F'[t]-tF''[t])))((-1+\beta)(2(-2+\beta)\\
&\times(-3+2\beta)+3t^{\beta}\beta(-4+3\beta)b_1)^2)+(t(9t^{2\beta}(4-3\beta)^2\beta^2b_1^2((-5+3\beta)\\
&\times((-4+3\beta)F'[t]-2tF''[t])+t^2F^{(3)}[t])+4(3-2\beta)^2(-2+\beta)^2\\
&\times(2(-5+2\beta)((-2+\beta)F'[t]-tF''[t])+t^2F^{(3)}[t])+6t^{\beta}(-2+\beta)\beta\\
&\times(-3+2\beta)(-4+3\beta)b_1((40+\beta(-45+11\beta))F'[t]+2t(-5(-2+\beta)\\
&\times F''[t]+tF^{(3)}[t]))))((-1+\beta)(2(-2+\beta)(-3+2\beta)+3t^{\beta}\beta\\
&\left.\times(-4+3\beta)b_1)^3)^{-1}\right)^{-1}.
\end{aligned}
$$

2.5.4. *Unification of matter dominated and accelerated phases*

For this framework, the Hubble rate and corresponding scale factor can be defined as follows [42, 44, 73, 74]

$$
H(t)=H_2+\frac{H_1}{t}\Rightarrow a(t)=b_3e^{H_2t}t^{H_1}. \tag{16}
$$

This corresponds to early Universe (for $t \ll t_0$) which implies $H(t)\sim\frac{H_1}{t}$ where the Universe was filled with perfect fluid with EoS parameter as $w=1+\frac{2}{3H_1}$. For $t\gg t_0\Rightarrow H\rightarrow H_0$ which seems to be de Sitter-like Universe. Also, the above form of $H(t)$ shows the transition from a matter dominated to the accelerating phase.

3. Reconstruction of Models and Cosmological Analysis

3.1. *For HDE with Hubble horizon*

By comparing the energy densities of this model HDE (12) and higher order corrected modified teleparallel gravity (5), we obtain the $F(t)$ models corresponding to four scale factor:

- **For power law scale factor:**
  For this scale factor, we have plotted $F(t)$ model versus cosmic time t as shown in Fig. 1 (left panel) with three different values of m and $c=1.21$. The reconstructed $F(t)$ model represents the increasing behavior as the cosmic time increases.

  The plot of EoS parameter has been presented in Fig. 1 (right panel). This parameter shows decreasing behavior as the cosmic time increases. This evolutes the Universe from vacuum DE era toward phantom era of the Universe by crossing the phantom divide line.

- **For bouncing scale factor:**
  The $F(t)$ model corresponding to this scale factor is plotted in Fig. 2 (left panel). It can be seen that this model also shows increasing behavior. The corresponding

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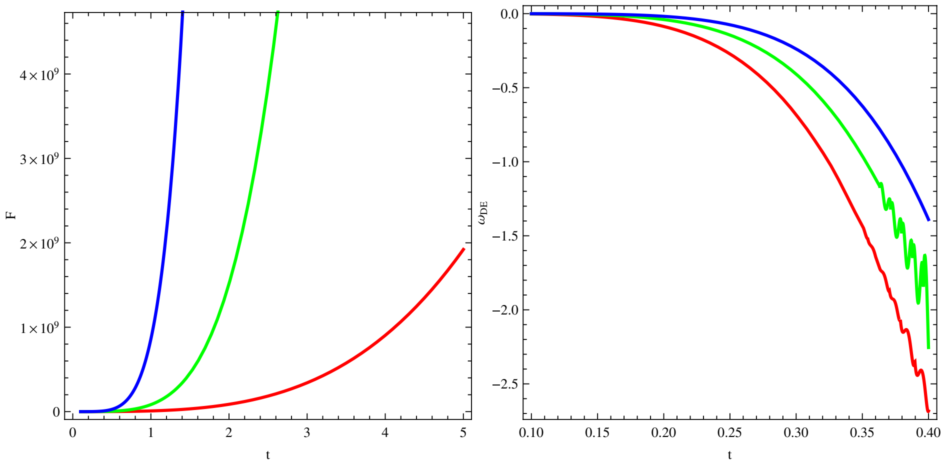

Fig. 1. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and power law scale factor with $m = 2.4$ (red), $m = 2.6$ (green) and $m = 2.8$ (blue).

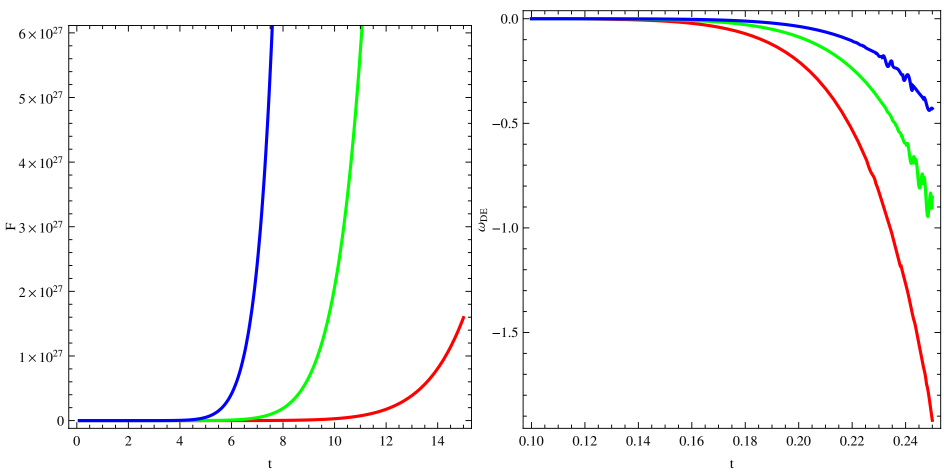

Fig. 2. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and bouncing scale factor with $n = 2.6$ (red), $n = 2.7$ (green) and $n = 2.8$ (blue).

EoS parameter is given in Fig. 2 (right panel) which shows quintessence, vacuum and phantom Universe for $m = 4$ and quintom-like behavior is being observed in this case. However, it represents the quintessence-like behavior for other two cases of m.

- **For intermediate scale factor:**
  The $F(t)$ model is plotted in Fig. 3 (left panel) which represents the increasing behavior as well as remains positive throughout the cosmic time for the cases

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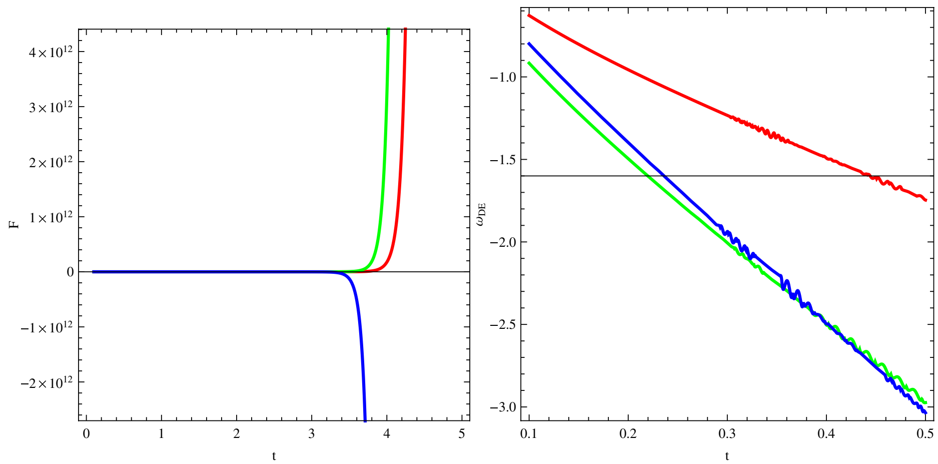

Fig. 3. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and intermediate scale factor with $\beta = 2.6$ (red), $\beta = 2.7$ (green) and $\beta = 2.8$ (blue).

$\beta = 2.6, 2.7$. However, the model shows decreasing behavior after some cosmic interval of time for the case $\beta = 2.8$. The corresponding EoS parameter is given in Fig. 3 (right panel). This parameter crosses the phantom divide line and also it evolutes the Universe from quintessence DE era toward phantom era.

- **For Hubble parameter form of scale factor:**
  The $F(t)$ model is plotted in Fig. 4 (left panel) which shows the increasing behavior as well as remains positive throughout the cosmic time for all values

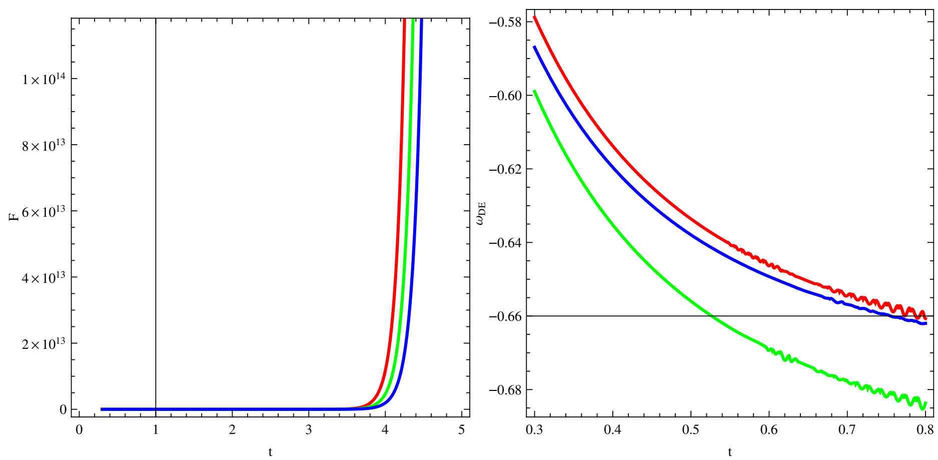

Fig. 4. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and Hubble parameter form of scale factor with $H_1 = 3.3$ (red), $H_1 = 3.4$ (green) and $H_1 = 3.5$ (blue).

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of H_1. However, EoS parameter evolutes the Universe in the quintessence region and is given in Fig. 4 (right panel).

3.2. *For HDE with event horizon*

As per previous subsection, we obtain the $F(t)$ models and corresponding EoS parameter by assuming three-scale factor. The detailed discussions about the results are as follows:

- **For power law scale factor:**
  $F(t)$ model versus cosmic time t for this scale factor is shown in Fig. 5 (left panel) and this reconstructed $F(t)$ model represents the decreasing behavior with the passage of time. Moreover, EoS parameter is shown in Fig. 5 (right panel) which is gradually decreasing as the cosmic time increases and represents the quintom-like behavior of the Universe. This evolutes the Universe from quintessence region to phantom by crossing the ΛCDM limit.
- **For intermediate scale factor:**
  For this scale factor, we have $F(t)$ model which is displayed in Fig. 6 (left panel), which represents the increasing behavior as well as remains positive throughout the cosmic time for the cases $\beta = 2.6,\ 2.7$. However, the model shows decreasing behavior after some cosmic interval of time for the case $\beta = 2.8$. The corresponding EoS parameter is given in Fig. 6 (right panel). This parameter crosses the phantom divide line and also it evolutes the Universe from quintessence DE era toward phantom era for all cases of β.
- **For Hubble parameter form of scale factor:**
  The $F(t)$ model is plotted in Fig. 7 (left panel) which indicates the decreasing behavior as well as remains negative after some interval of time for all values

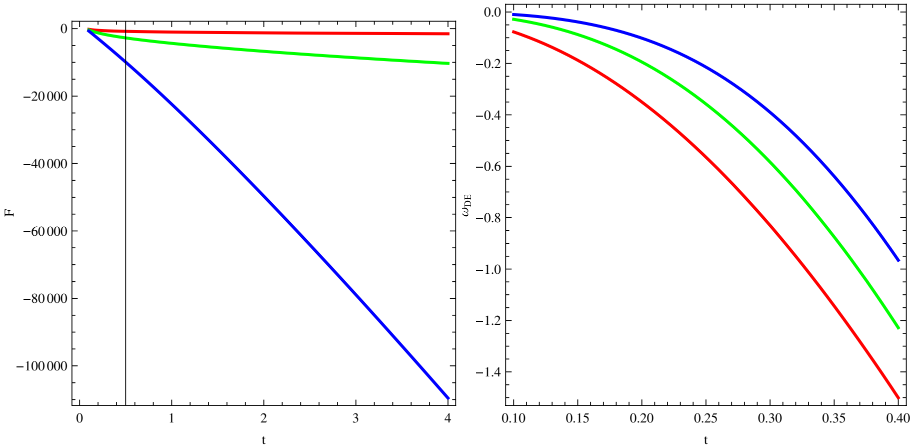

Fig. 5. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with event horizon and power law scale factor with $m = 1.4$ (red), $m = 1.6$ (green) and $m = 1.8$ (blue).

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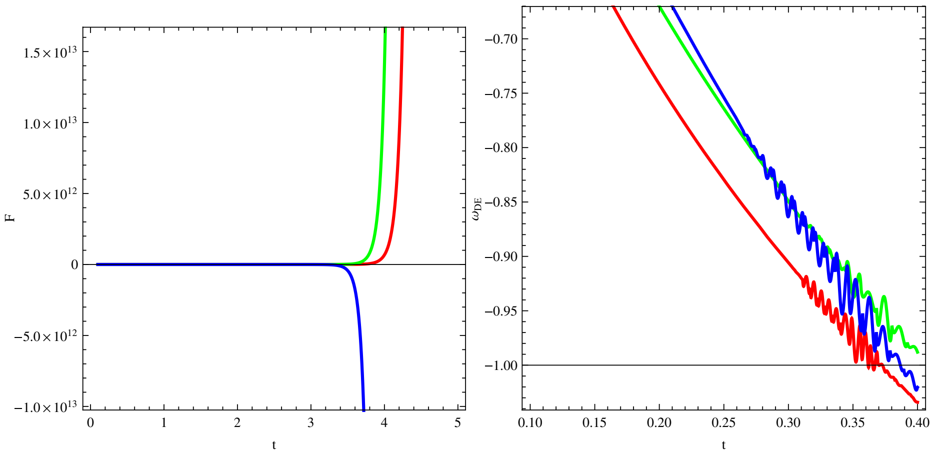

Fig. 6. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with event horizon and intermediate scale factor with $\beta = 2.6$ (red), $\beta = 2.7$ (green) and $\beta = 2.8$ (blue).

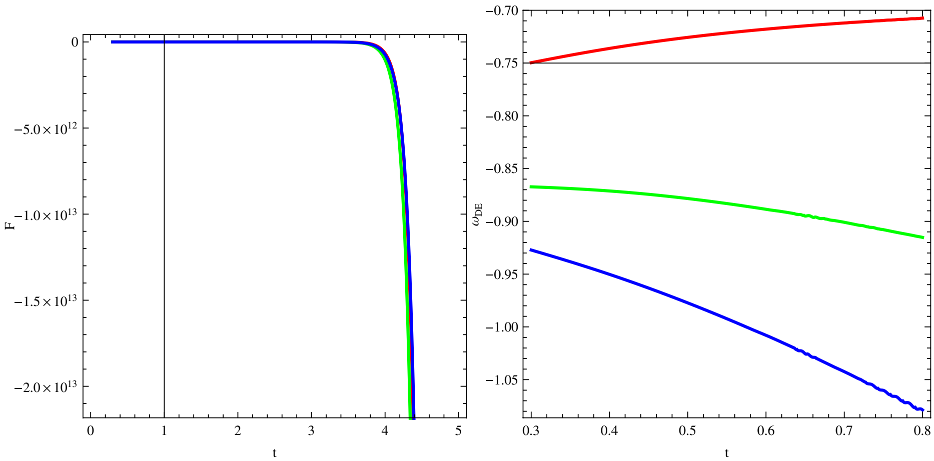

Fig. 7. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with event horizon and Hubble parameter of scale factor with $H_1 = 3.3$ (red), $H_1 = 3.4$ (green) and $H_1 = 3.5$ (blue).

of H_1. However, EoS parameter evolutes the Universe in the quintessence region for $H_1 = 3.3,\ 3.4$ and is given in Fig. 7 (right panel). For $H_1 = 3.5$, the EoS parameter exhibits the quintom-like nature.

3.3. *For NHDE*

In the similar way, one can attain the $F(t)$ models and corresponding EoS parameter for four scale factors for NHDE. The detailed discussions about the results are

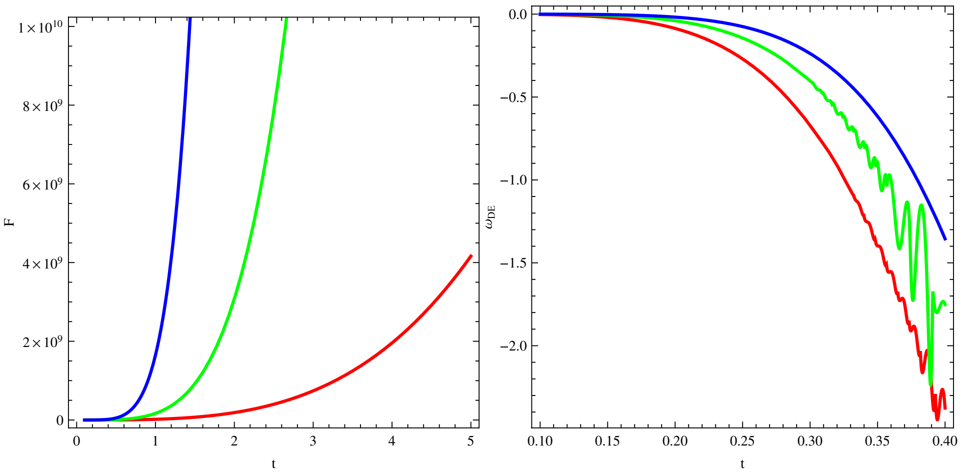

Fig. 8. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and power law scale factor with $m = 2.4$ (red), $m = 2.6$ (green) and $m = 2.8$ (blue).

as follows:

- **For power law scale factor:**
  For this scale factor, $F(t)$ model versus cosmic time t as shown in Fig. 8 (left panel) with same constant parameters as mentioned above. The reconstructed $F(t)$ model represents the increasing behavior and remains positive for all values of m throughout cosmic time. The plot of EoS parameter has been presented in

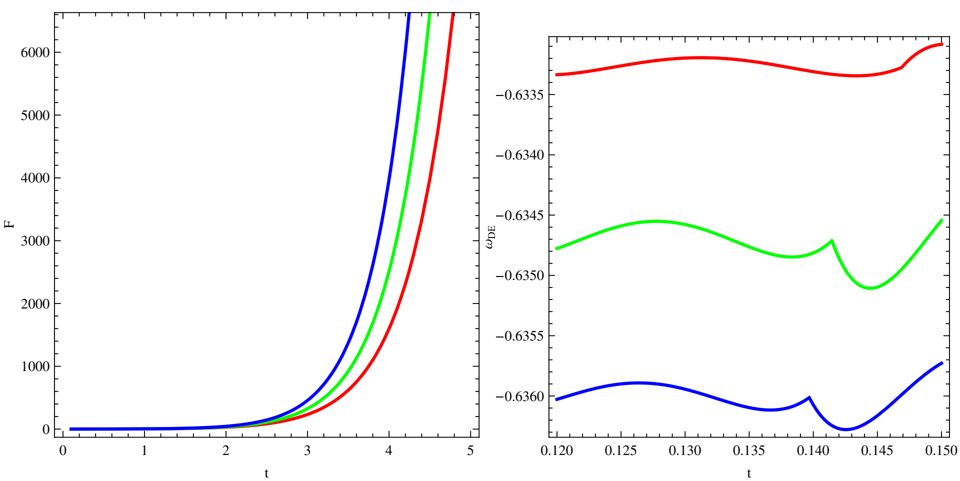

Fig. 9. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and bouncing scale factor with $n = 2.6$ (red), $n = 2.7$ (green) and $n = 2.8$ (blue).

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Fig. 8 (right panel). The EoS analysis shows that the Universe is driven to the phantom region from vacuum DE era toward phantom era of the Universe by crossing the phantom divide line.

- **For bouncing scale factor:**
  Figure 9 (left panel) represents that the reconstructed $F(t)$ model remains positive as well as increasing as the cosmic time increases. The corresponding EoS

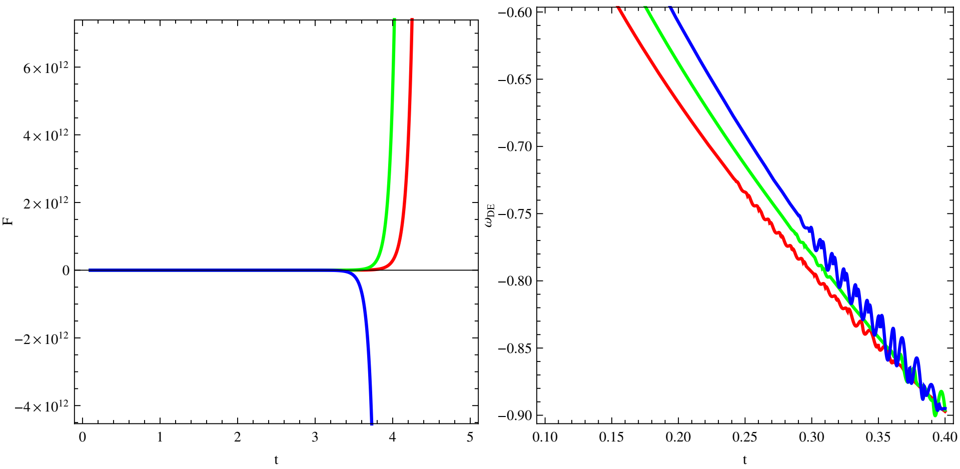

Fig. 10. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and intermediate scale factor with $\beta = 2.6$ (red), $\beta = 2.7$ (green) and $\beta = 2.8$ (blue).

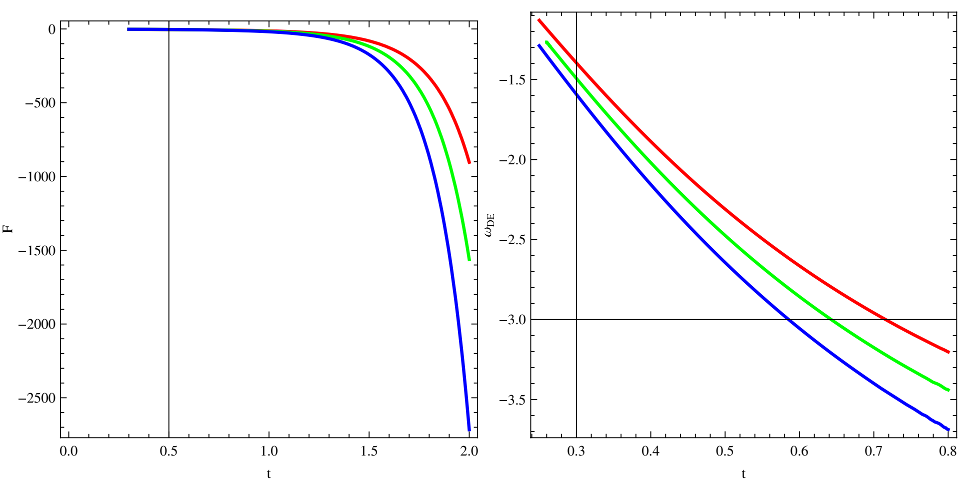

Fig. 11. (Color online) Plots of $F(t)$ (left) and $\omega_{\rm DE}$ (right) versus t (in seconds) for HDE with Hubble horizon and Hubble parameter form of scale factor with $H_1 = 3.3$ (red), $H_1 = 3.4$ (green) and $H_1 = 3.5$ (blue).

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parameter is given in Fig. 9 (right panel) which shows the quintessence-like behavior for all cases of m.

- **For intermediate scale factor:**
The $F(t)$ model is plotted in Fig. 10 (left panel) which represents the increasing behavior as well as remains positive throughout the cosmic time for the cases $\beta = 2.6,\ 2.7$. However, the model shows decreasing behavior after some cosmic interval of time for the case $\beta = 2.8$. The corresponding EoS parameter is given in Fig. 10 (right panel) which evolutes the Universe in quintessence DE era.
- **For Hubble parameter form of scale factor:**
The $F(t)$ model is plotted in Fig. 11 (left panel) which shows the decreasing behavior as well as remains negative for all values of H_1. However, EoS parameter evolutes the Universe in the phantom region and is given in Fig. 11 (right panel).

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4. Concluding Remarks

Recently, Otalora and Saridakis [41] have developed the novel torsional gravitational modifications using higher derivative, $(\nabla T)^2$ and $\Box T$ terms, i.e. theories that are characterized by the Lagrangian $F(T, (\nabla T)^2, \Box T)$. They have investigated various cosmological parameters by choosing specific models of this gravity. They have suggested that DE EoS parameter lies in the quintessence regime, or may exhibit the phantom divide crossing during the cosmological evolution. Finally, the scale factor behaves asymptotically either as a power law or as an exponential, in agreement with observations. In this paper, we have considered this gravity and discussed its various aspects by assuming DE models (HDE with Hubble and event horizons, NHDE) through reconstruction scenario.

For obtaining the reconstructed models, we have assumed four scale factors named as power law, bouncing, intermediate and Hubble parameter form. We have obtained the $F(t)$ models corresponding to above scenario numerically and displayed them in graphs. The reconstructed $F(t)$ models corresponding to each model can be concluded as follows: The $F(t)$ models in case of HDE model with Hubble horizon corresponding to four scale factors are shown in the left panel of Figs. 1–4. In this case of HDE, $F(t)$ models exhibit the increasing behavior with the passage of time and show the variation with respect to constant parameters. For HDE with event horizon, $F(t)$ models (as shown in left panel of Figs. 5–7) show decreasing behavior for power law scale factor and Hubble parameter form of scale factor. However, it illustrates the increasing behavior with the passage of time for intermediate scale factor. For NHDE case, $F(t)$ models (as shown in left panel of Figs. 8–11) indicate the increasing behavior for power law and bouncing and in two cases of intermediate scale factors. However, it exhibits the decreasing behavior for last case of scale factor.

We have also investigated the EoS parameter corresponding to all reconstructed models. The results have been summarized as follows: For HDE with Hubble horizon, the EoS parameter crosses the phantom divided line from quintessence to phantom by evolving the vacuum region for power law, bouncing, intermediate (right panels of Figs. 1–3). However, in case of Hubble parameter form (right panel of Fig. 4), this parameter only lies in the quintessence region. For HDE with event horizon, the EoS has started from quintessence region and goes toward by crossing the ΛCDM limit in all cases of scale factors (right panels of Figs. 5–7). For NHDE in power law form of scale factor, the EoS parameter evolutes the Universe from vacuum DE era toward phantom era of the Universe by crossing the phantom divide line (right panel of Fig. 8). The EoS parameter in case of bouncing scale factor shows the quintessence-like behavior for all cases of m (right panel of Fig. 9). The EoS parameter, in intermediate scale factor case, evolutes the Universe in quintessence DE era (right panel of Fig. 10). In Hubble parameter form of scale factor, EoS parameter evolutes the Universe in the phantom region and is given in Fig. 11 (right panel). In view of above discussions, we have observed that the EoS parameter provides quintom-like nature of the Universe in most of the cases, i.e. it evolutes the Universe from vacuum DE era toward phantom era of the Universe by crossing the phantom divide line. In most cases of HDE models and scale factor choices, the EoS parameter meets the ΛCDM limit (vacuum region) which corresponds to cosmological constant, i.e. $\omega_{\mathrm{DE0}} = -1$.

Moreover, the ranges of EoS parameter lie within the observational constraints as mentioned by Ade *et al.* [75] (Planck data) as

$$
\begin{aligned}
&\omega_{\mathrm{DE}} = -1.13^{+0.24}_{-0.25} && \text{(Planck+WP+BAO)},\\
&\omega_{\mathrm{DE}} = -1.09 \pm 0.17 && \text{(Planck+WP+Union 2.1)},\\
&\omega_{\mathrm{DE}} = -1.13^{+0.13}_{-0.14} && \text{(Planck+WP+SNLS)},\\
&\omega_{\mathrm{DE}} = -1.24^{+0.18}_{-0.19} && \text{(Planck+WP+}H_0\text{)}.
\end{aligned}
$$

The trajectories also favor the nine-year WMAP observational data [76] which give the ranges for EoS parameter as

$$
\begin{aligned}
&\omega_{\mathrm{DE}} = -1.073^{+0.090}_{-0.089} && \text{(WMAP+eCMB+BAO+}H_0\text{)},\\
&\omega_{\mathrm{DE}} = -1.084 \pm 0.063 && \text{(WMAP+eCMB+BAO+}H_0\text{+SNe)}.
\end{aligned}
$$

The above constraints have been obtained by implying different combination of observational schemes at 95% confidence level.

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