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A GENERAL CLASS OF ESTIMATOR OF POPULATION MEAN IN PRESENCE OF NON–RESPONSE

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ABSTRACT

In this paper we have proposed some estimators of population mean in case of non–response using information of a single auxiliary variable. A general estimator in case of non–response has also been proposed. The expressions for mean square errors of proposed estimators have been derived. Special cases of proposed estimator have been obtained using different values of constants.

KEY WORDS

Non–response, Mean Square Error, Auxiliary Variable.

1. INTRODUCTION

Non-response as an aspect in almost every type of sample survey creates problems for estimation. Furthermore this cannot be eliminated by simply increasing sample size. The presence of non-response distorts parameter estimation by increasing bias in estimates resulting in larger mean square error. Non-response can be classified as ignorable or non-ignorable depending on whether it is correlated with the target variable [Little, 1982 and Glynn, Laird and Rubin, 1993]. The non–response always exists when surveying human populations as people hesitate to respond in surveys; and increases notably while studying sensitive issues. It has been observed that presence of non–response increases the bias in estimates; ultimately reducing their efficiency. Many survey statisticians have proposed methods for estimating population characteristics in presence of non–response. The sub-sampling method has been a popular method in case of non–response. We now discuss this method alongside some popular estimators when non–response occurred in sample surveys.

Suppose a simple random sample without replacement (SRSWOR) of size n is drawn from a population of size N. From the available sample, r_1 units respond to survey variable Y and r_2 units do not respond. Corresponding to sample respondents and non–respondents, the population is also divided in same sort of groups containing N_1 and N_2 units. Out of r_2 non–respondents, a sub-sample of k ($k=r_2/h$, $h>1$) units is drawn and information is obtained from these k units. Hansen and Hurwitz (1946) suggested following estimator of population mean when sub-sampling is used to overcome non–response:

$$\bar{y}^* = (r_1/n)\bar{y}_{r_1 1} + (r_2/n)\bar{y}_{k2}\,, \tag{1.1}$$

where $\bar{y}_1 = r_1^{-1}\sum_{i=1}^{r_1} y_i$ and $\bar{y}_{k2} = k^{-1}\sum_{i=1}^{k} y_i$ are means of variable of interest. The estimator (1.1) is unbiased with variance:

$$Var\left(\bar{y}^*\right) = \lambda S_y^2 + \theta S_{y_2}^2\,, \tag{1.2}$$

with $S_y^2 = \sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2 \Big/ (N-1)$, $S_{y_2}^2 = \sum_{i=1}^{N_2}\left(y_i - \bar{Y}_2\right)^2 \Big/ (N_2 - 1)$, $\lambda = (1-f)/n$, $f = n/N$, $\theta = W_2(h-1)/n$, $W_2 = N_2/N$, $\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$ and $\bar{Y}_2 = N_2^{-1}\sum_{i=1}^{N_2} y_i$. Some notable references are of Cochran (1977), Rao (1986), Naik and Gupta (1991), Tripathi and Khare (1997), Tabasum and Khan (2006) and Singh and Kumar (2008a, 2008b, 2008c, 2009)

2. NON–RESPONSE IN TWO PHASE SAMPLING

The two phase sampling procedure has been effectively used in case of non–response to increase the precision of estimates. The two phase sampling procedure in case of non–response is described as:

i) Select a first phase sample of size n_1 using SRSWOR and record information on auxiliary variable X.
ii) Select a second phase sample of size n_2 using SRSWOR from first phase sample of size n_1. The r_1 units respond and r_2 units do not respond. Collect information on study variable Y from responding units.
iii) Select a subsample of size k ($k=r_2/h$, $h>1$) and record information on study variable from these selected units.

Using two phase sampling procedure, various authors have proposed different estimators of population mean in case of non–response. The estimators have been proposed by considering following two situations:

Situation 1:

Unknown population mean of auxiliary variable X, incomplete information on Y and X.

Situation 2:

Unknown population mean of auxiliary variable X, incomplete information on Y and complete information on X.

Khare and Srivastava (1993, 1995) and Tabasum and Khan (2004) proposed following ratio, product and regression estimators for Situation 1:

$$t_{R1d} = \bar{y}^*\left(\bar{x}_1/\bar{x}^*\right),\ t_{P1d} = \bar{y}^*\left(\bar{x}^*/\bar{x}_1\right),\ t_{LR1d} = \bar{y}^* + b^*\left(\bar{x}_1 - \bar{x}^*\right).$$

The mean square errors of above estimators; upto first degree approximation; are:

$$Var\left(t_{R1d}\right) = \lambda_3\left\{S_y^2 + R^2 S_x^2(1-2C)\right\} + \lambda_1 S_y^2 + \theta\left\{S_{y_2}^2 + R^2 S_{x_2}^2\left(1 - 2C_{(2)}\right)\right\}, \tag{2.1}$$

$$Var\left(t_{P1d}\right)=\lambda_3\left\{S_y^2+R^2S_x^2\left(1+2C\right)\right\}+\lambda_1S_y^2+\theta\left\{S_{y_2}^2+R^2S_{x_2}^2\left(1+2C_{(2)}\right)\right\}, \quad (2.2)$$

and

$$Var\left(t_{LR1d}\right)=\lambda_3\left(1-\rho^2\right)S_y^2+\lambda_1S_y^2+\theta\left\{S_{y_2}^2+R^2CS_{x_2}^2\left(C-2C_{(2)}\right)\right\}, \quad (2.3)$$

where

$$\lambda_1=n_1^{-1}-N^{-1},\ \lambda_3=n_2^{-1}-n_1^{-1},\ R=\bar{Y}/\bar{X},\ C=\beta/R,\ \beta=S_{xy}/S_x^2,$$

$$C_{(2)}=\beta_{(2)}/R,\beta_{(2)}=S_{xy(2)}/S_{x_2}^2,\ S_x^2=\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2/\left(N-1\right)$$

$$S_{x_2}^2=\sum_{i=1}^{N_2}\left(x_i-\bar{X}_2\right)^2/\left(N-1\right),\ \bar{X}_2=N_2^{-1}\sum_1^{N_2}x_i,$$

$$S_{xy}=\sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(y_i-\bar{Y}\right)/\left(N-1\right)\text{ and }S_{xy(2)}=\sum_{i=1}^{N_2}\left(x_i-\bar{X}_2\right)\left(y_i-\bar{Y}_2\right)/\left(N_2-1\right).$$

Singh et al. (2010) proposed following exponential type estimators under Situation 1:

$$t_{R1d(SKK)}=\bar{y}^*\exp\left\{\frac{\bar{x}_1-\bar{x}^*}{\bar{x}_1+\bar{x}^*}\right\},\ t_{P1d(SKK)}=\bar{y}^*\exp\left\{\frac{\bar{x}^*-\bar{x}_1}{\bar{x}^*+\bar{x}_1}\right\}.$$

Singh et al. (2010) derived the variances of above two estimators i.e.

$$Var\left(t_{R1d(SKK)}\right)=\lambda_3\left\{S_y^2+\left(R^2S_x^2/4\right)\left(1-4C\right)\right\}+\lambda_1S_y^2+\theta\left\{S_{y_2}^2+\left(R^2S_{x_2}^2/4\right)\left(1-4C_{(2)}\right)\right\}, \quad (2.4)$$

and

$$Var\left(t_{P1d(SKK)}\right)=\lambda_3\left\{S_y^2+\left(R^2S_x^2/4\right)\left(1+4C\right)\right\}+\lambda_1S_y^2+\theta\left\{S_{y_2}^2+\left(R^2S_{x_2}^2/4\right)\left(1+4C_{(2)}\right)\right\}. \quad (2.5)$$

respectively

Khare and Srivastava (1993, 1995) and Singh et al. (2010) have also proposed estimators for Situation 2. The exponential estimator, proposed by Singh et al. (2010) for situation 2 was

$$t_{R2d(SKK)}=\bar{y}^*\exp\left\{\frac{\bar{x}_1-\bar{x}}{\bar{x}_1+\bar{x}}\right\},\ t_{P2d(SKK)}=\bar{y}^*\exp\left\{\frac{\bar{x}-\bar{x}_1}{\bar{x}+\bar{x}_1}\right\}.$$

$$Var\left(t_{R2d(SKK)}\right)=\lambda_2S_y^2+\theta S_{y_2}^2+\lambda_1\left(R^2S_x^2/4\right)\left(1-4C\right), \quad (2.6)$$

and

$$Var\left(t_{P2d(SKK)}\right)=\lambda_2S_y^2+\theta S_{y_2}^2+\lambda_1\left(R^2S_x^2/4\right)\left(1+4C\right), \quad (2.7)$$

respectively, where $\lambda_2=n_2^{-1}-N^{-1}$.

In the following we propose some estimators of population mean for two phase sampling in case of non–response.

3. THE PROPOSED ESTIMATORS

In this section we have suggested some new estimators of population mean for two phase sampling in case of non–response. The estimators have been suggested for two different situations discussed in section 2. These estimators are given in next section.

3.1 Proposed Estimators for Situation 1

We have described situation 1 in section 2 as a situation when incomplete information is available on auxiliary variable and study variable. We now propose following ratio type estimators for this situation:

$$t_{1n} = \bar{y}^* \left\{ \frac{\bar{x}^* - w\left(\bar{x}^* - \bar{x}_1\right)}{\bar{x}_1 + w\left(\bar{x}^* - \bar{x}_1\right)} \right\}, \tag{3.1}$$

where w is a constant. Using $\bar{y}^* = \bar{Y} + \bar{e}_y^*$, $\bar{x}^* = \bar{X} + \bar{e}_x^*$ and $\bar{x}_1 = \bar{X} + \bar{e}_{x_1}$ the above estimator can be written as:

$$t_{1n} - \bar{Y} = \bar{e}_y^* - \left(2w-1\right)R\bar{e}_x^* + \left(2w-1\right)R\bar{e}_{x_1} .$$

Squaring above equation, applying expectation and using following results:

$$\left.\begin{array}{l} E\left(\bar{e}_y^{*2}\right) = \lambda_2 S_y^2 + \theta S_{y_2}^2 \ ; \ E\left(\bar{e}_x^{*2}\right) = \lambda_2 S_x^2 + \theta S_{x_2}^2 \ ; E\left(\bar{e}_{x_1}^2\right) = \lambda_1 S_x^2 \\ E\left(\bar{e}_y^* \bar{e}_x^*\right) = \lambda_2 S_{xy} + \theta S_{xy(2)} \ ; E\left(\bar{e}_y^* \bar{e}_{x_1}\right) = \lambda_1 S_{xy} \ ; E\left(\bar{e}_x^* \bar{e}_{x_1}\right) = \lambda_1 S_x^2 \end{array}\right\}, \tag{3.2}$$

the variance of (3.1) can be written as:

$$Var\left(t_{1n}\right) = \lambda_3 \left[S_y^2 + R^2 S_x^2 \left\{ \left(2w-1\right)^2 - 2\left(2w-1\right)C \right\} \right] + \lambda_1 S_y^2$$
$$+ \theta \left[S_{y_2}^2 + R^2 S_{x_2}^2 \left\{ \left(2w-1\right)^2 - 2\left(2w-1\right)C_{(2)} \right\} \right]. \tag{3.3}$$

Immediate comparison of (3.3) with (2.1) shows that the proposed estimator $t_{21d(ISH)}$ will be more precise as compared with Khare and Srivastava (1993) estimator is $C < w < 1$ and $C_{(2)} < w < 1$.

3.2 Proposed Estimators for Situation 2

We now propose following estimator for situation 2:

$$t_{2n} = \bar{y}^* \left\{ \frac{\bar{x} - w\left(\bar{x} - \bar{x}_1\right)}{\bar{x}_1 + w\left(\bar{x} - \bar{x}_1\right)} \right\}, \tag{3.4}$$

where w is a constant. Using the relations $\bar{y}^* = \bar{Y} + \bar{e}_y^*$, $\bar{x} = \bar{X} + \bar{e}_x$ and $\bar{x}_1 = \bar{X} + \bar{e}_{x_1}$ we can write (3.4) as:

$$t_{2n} - \bar{Y} = \bar{e}_y^* - (2w-1)R\bar{e}_x + (2w-1)R\bar{e}_{x_1}.$$

Squaring above equation, applying expectation and using (3.2), the variance of (3.4) is:

$$Var(t_{2n}) = \lambda_3\left[S_y^2 + R^2S_x^2\left\{(2w-1)^2 - 2(2w-1)C\right\}\right] + \lambda_1 S_y^2 + \theta S_{y_2}^2 \tag{3.5}$$

Khare and Srivastava (1993) proposed a ratio type estimator for situation 2. The variance of their proposed estimator is:

$$Var(t_{2n}) = \lambda_3\left[S_y^2 + R^2S_x^2(1-2C)\right] + \lambda_1 S_y^2 + \theta S_{y_2}^2. \tag{3.6}$$

Immediate comparison of (3.5) with (3.6) shows that the proposed estimator $t_{21d(ISH)}$ will be more precise as compared with Khare and Srivastava (1993) estimator if $w > 1-C$.

We now propose general estimators of population mean in two phase sampling in presence of non–response.

4. GENERALIZED ESTIMATORS

We now propose general estimators of population mean for two phase sampling when there exist non–response for study variable *Y*. We have proposed two estimators for two different situations discussed above. The estimators are proposed in the following.

4.1 The General Estimator-1

We propose following general estimator of population mean under Situation 1:

$$T_{1g} = \bar{y}^*\left\{\frac{\bar{x}_1 + a\left(\bar{x}^* - \bar{x}_1\right)}{\bar{x}_1 + b\left(\bar{x}^* - \bar{x}_1\right)}\right\}^p \tag{4.1}$$

Using the transformation $\bar{y}^* = \bar{Y} + \bar{e}_y^*$, $\bar{x}^* = \bar{X} + \bar{e}_x^*$ and $\bar{x}_1 = \bar{X} + \bar{e}_{x_1}$ in (4.1) and expanding upto first order term, we have:

$$T_{1g} - \bar{Y} = \bar{e}_y^* - p(a-b)R\bar{e}_{x_1} + p(a-b)R\bar{e}_x^*,$$

or

$$T_{1g} - \bar{Y} = \bar{e}_y^* + DR\bar{e}_{x_1} - DR\bar{e}_x^*,$$

where $D = p(b-a)$. Squaring above equation and applying expectation, the variance of T_{1g} is:

$$\begin{aligned} Var(T_{1g}) = {} & E\left(\bar{e}_y^{*2}\right) + D^2R^2E\left(\bar{e}_{x_1}^2\right) + D^2R^2E\left(\bar{e}_x^{*2}\right) + 2DRE\left(\bar{e}_y^*\bar{e}_{x_1}\right) \\ & -2DRE\left(\bar{e}_y^*\bar{e}_x^*\right) - 2D^2R^2E\left(\bar{e}_{x_1}\bar{e}_x^*\right) \end{aligned} \tag{4.2}$$

Now using (3.2) in (4.2), the variance of T_{1d} becomes:

$$Var\left(T_{1g}\right) = \lambda_2 S_y^2 + D^2R^2\lambda_1 S_x^2 - D^2R^2\left(\lambda_2 S_x^2 + \theta S_{x_2}^2\right) \\ +2DR\lambda_1 S_{xy} + 2DR\left(\lambda_2 S_{xy} + \theta S_{xy_2}\right) - 2D^2R^2\lambda_1 S_x^2 + \theta S_{y_2}^2$$

Simplifying above equation, the variance of T_{1g} is given as:

$$Var\left(T_{1g}\right) = \lambda_3\left[S_y^2 + R^2S_x^2\left(D^2 - 2CD\right)\right] + \lambda_1 S_y^2 + \theta\left[S_{y_2}^2 + R^2S_{x_2}^2\left(D^2 - 2C_{(2)}D\right)\right], \tag{4.3}$$

where $D = p\left(b-a\right)$. Comparing (4.3) with (2.4) we can see that the proposed estimator is more efficient as compared with the estimator proposed by Singh et al. (2010) if $\left(4C-1\right)/2 \le D \le 1/2$ and $\left(4C_{(2)}-1\right)/2 \le D \le 1/2$. We have also given special cases of proposed estimator (4.1) in Appendix 1.

4.2 The General Estimator-2

We propose following general estimator of population mean in two phase sampling with non–response for Situation 2 as under:

$$T_{2g} = \bar{y}^*\left\{\frac{\bar{x}_1 + a\left(\bar{x} - \bar{x}_1\right)}{\bar{x}_1 + b\left(\bar{x} - \bar{x}_1\right)}\right\}^p . \tag{4.4}$$

Using the transformation $\bar{y}^* = \bar{Y} + \bar{e}_y^*$, $\bar{x} = \bar{X} + \bar{e}_x$ and $\bar{x}_1 = \bar{X} + \bar{e}_{x_1}$, the above estimator can be written as:

$$T_{2g} - \bar{Y} = \bar{e}_y^* - p\left(a-b\right)R\bar{e}_{x_1} + p\left(a-b\right)R\bar{e}_x = \bar{e}_y^* + DR\bar{e}_{x_1} - DR\bar{e}_x ,$$

where D is defined earlier. Squaring above equation and applying expectation we have:

$$Var\left(T_{2g}\right) = E\left(\bar{e}_y^{*2}\right) + D^2R^2E\left(\bar{e}_{x_1}^2\right) + D^2R^2E\left(\bar{e}_x^2\right) + 2DRE\left(\bar{e}_y^*\bar{e}_{x_1}\right) \\ -2DRE\left(\bar{e}_y^*\bar{e}_x\right) - 2D^2R^2E\left(\bar{e}_{x_1}\bar{e}_x\right) \tag{4.5}$$

Using (3.2) in (4.5), the variance of T_{2g} becomes:

$$Var\left(T_{2g}\right) = \lambda S_y^2 + D^2R^2\lambda_1 S_x^2 + D^2R^2\lambda_2 S_x^2 + 2DR\lambda_1 S_{xy} - 2DR\lambda_2 S_{xy} - 2D^2R^2\lambda_1 S_{xy} + \theta S_{y_2}^2$$

Simplifying above equation, the variance of T_{2d} is given as:

$$Var\left(T_{2g}\right) = \lambda_3\left[S_y^2 + R^2S_x^2\left(D^2 - 2CD\right)\right] + \lambda_1 S_y^2 + \theta S_{y_2}^2 . \tag{4.6}$$

The proposed estimator (4.4) will be more efficient than Khare and Srivastava (1993) estimator if $2C - 1 \le D \le 1$.

5. OPTIMUM SAMPLE SIZE UNDER LINEAR COST FUNCTION

In this section we obtaine the optimum values of sample sizes at various phases under the linear cost function. We consider following linear cost function for this purpose:

$$C = c_1 n_1 + c_2 n_2 + c_3 r_1 + c_4 k\,, \tag{5.1}$$

where c_1 is unit cost associated with first phase sample, c_2 is unit cost for second phase sample, c_3 is unit cost for first sample on Y and c_4 is unit cost associated with response variable for sub sample of size k. The expected cost for obtaining information is:

$$C^* = E\left(C\right) = c_1 n_1 + n_2\left(c_2 + c_3 W_1 + h^{-1} c_4 W_2\right). \tag{5.2}$$

Our objective now is to minimize (5.2) subject to the condition that (4.3) is equal to a fixed quantity V_0. We use the Langrangian technique to achieve this task. The function to be optimized is:

$$\phi = C^* + \lambda\left\{Var\left(T_{1d}\right) - V_0\right\}$$

or

$$\phi = c_1 n_1 + n_2\left(c_2 + c_3 W_1 + h^{-1} c_4 W_2\right) + \delta\left\{\left(n_1^{-1} - N\right)S_y^2 + \left(n_2^{-1} - n_1^{-1}\right)S_r^2 + n_2^{-1} W_2\left(h-1\right)S_{r2}^2 - V_0\right\}, \tag{5.3}$$

where $S_r^2 = S_y^2 + R^2 S_x^2\left(D^2 - 2CD\right)$ and $S_{r2}^2 = S_{y_2}^2 + R^2 S_{x_2}^2\left(D^2 - 2C_{(2)}D\right)$. The optimum values of n_1, n_2 and h are obtained by differentiating (5.3). The partial derivatives of (5.3) with respect to n_1, n_2, h and δ are:

$$\frac{\partial\phi}{\partial\delta} = \left(n_1^{-1} - N\right)S_y^2 + \left(n_2^{-1} - n_1^{-1}\right)S_r^2 + n_2^{-1} W_2\left(h-1\right)S_{r2}^2 - V_0\,,$$

$$\frac{\partial\phi}{\partial h} = -\frac{n_2 c_4 W_2}{h^2} + \frac{\delta W_2}{n_2} S_{r2}^2\,,$$

$$\frac{\partial\phi}{\partial n_1} = c_1 - \frac{\delta}{n_1^2}\left(S_y^2 - S_r^2\right),$$

and

$$\frac{\partial\phi}{\partial n_2} = c_2 + c_3 W_1 + \frac{c_4 W_2}{h} - \delta\left\{\frac{S_r^2}{n_2^2} + \frac{W_2\left(h-1\right)}{n_2^2} S_{r2}^2\right\}.$$

Now setting above partial derivatives equal to zero we have:

$$n_1^{-1}\left(S_y^2 - S_r^2\right) + n_2^{-1}\left\{S_r^2 + W_2\left(h-1\right)S_{r2}^2\right\} = V_0 + N^{-1} S_y^2\,, \tag{5.4}$$

$$h^2 = \frac{n_2^2 c_4}{\delta S_{r2}^2}, \tag{5.5}$$

$$n_1 = \sqrt{c_1^{-1}\delta\left(S_y^2 - S_r^2\right)}, \tag{5.6}$$

and

$$n_2 = \sqrt{\frac{\delta\left\{S_r^2 + W_2\left(h-1\right)S_{r2}^2\right\}}{c_2 + c_3 W + h^{-1} c_4 W_2}}. \tag{5.7}$$

Using (5.7) in (5.5), the optimum value of h is:

$$h = \frac{c_4\left\{S_r^2 + W_2\left(h-1\right)S_{r2}^2\right\}}{S_{r2}^2\left(c_2 + c_3 W + h^{-1} c_4 W_2\right)}. \tag{5.8}$$

Further, by using (5.6) in (5.4) and simplifying, we have:

$$\sqrt{\delta} = \frac{\sqrt{c_1\left(S_y^2 - S_r^2\right)} + \sqrt{\left(c_2 + c_3 W + h^{-1} c_4 W_2\right)\left(S_r^2 + W_2\left(h-1\right)S_{r2}^2\right)}}{V_0 + S_y^2/N}. \tag{5.9}$$

The optimum values of n_1 and n_2 can be obtained by using (5.9) in (5.6) and (5.7).

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Appendix A
Special Cases of Proposed Estimators

The Proposed Estimator 1			
Sr. No.	**Values of Constants**	**Estimator**	**Reference**
1.	$p = 0$	$T_{1d} = \bar{y}^*$	Hansen and Hurwitz (1946)
2.	$p = 1, a = 0, b = 1$	$T_{1d} = \bar{y}^* \left(\bar{x}_1 / \bar{x}^*\right)$	Khare and Srivastava (1993)
3.	$p = 1, a = 1, b = 0$	$T_{1d} = \bar{y}^* \left(\bar{x}^* / \bar{x}_1\right)$	Khare and Srivastava (1993)
4.	$p = p, a = 0, b = 1$	$T_{1d} = \bar{y}^* \left(\bar{x}_1 / \bar{x}^*\right)^p$	
5.	$p = 1, a = 1–w, b = w$	$T_{1d} = \bar{y}^* \left\{\dfrac{\bar{x}^* - w\left(\bar{x}^* - \bar{x}_1\right)}{\bar{x}_1 + w\left(\bar{x}^* - \bar{x}_1\right)}\right\}$	Equation (3.1)
The Proposed Estimator 1			
1.	$p = 0$	$T_{1d} = \bar{y}^*$	Hansen and Hurwitz (1946)
2.	$p = 1, a = 0, b = 1$	$T_{1d} = \bar{y}^* \left(\bar{x}_1 / \bar{x}\right)$	Khare and Srivastava (1993)
3.	$p = 1, a = 1, b = 1$	$T_{1d} = \bar{y}^* \left(\bar{x} / \bar{x}_1\right)$	Khare and Srivastava (1993)
4.	$p = p, a = 0, b = 1$	$T_{1d} = \bar{y}^* \left(\bar{x}_1 / \bar{x}\right)^p$	
5.	$p = 1, a = 1–w, b = w$	$T_{1d} = \bar{y}^* \left\{\dfrac{\bar{x} - w\left(\bar{x} - \bar{x}_1\right)}{\bar{x} + w\left(\bar{x} - \bar{x}_1\right)}\right\}$	Equation (3.4)