DISPERSIVE AND DISSIPATIVE BEHAVIOUR OF THE SPECTRAL ELEMENT METHOD

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Abstract. If the nodes for the spectral element method are chosen to be the Gauss-Legendre-Lobatto points, then the resulting mass matrix is diagonal and the method is sometimes then described as the *Gauss-point mass lumped finite element scheme*. We study the dispersive behaviour of the scheme issue in detail and provide both a qualitative description of the nature of the dispersive and dissipative behaviour of the scheme along with precise quantitative statements of the accuracy in terms of the mesh size and the order of the scheme. We prove that (a) the Gauss-point mass lumped scheme (i.e. spectral element method) tends to exhibit *phase lag* whereas the (consistent) finite element scheme tends to exhibit *phase lead*; (b) the absolute accuracy of the spectral element scheme is $1/p$ times better than that of the finite element scheme despite the use of numerical integration; (c) when the order p, the mesh-size h and the frequency of the wave ω satisfy $2p + 1 \approx \omega h$ the true wave is essentially fully resolved. As a consequence, one obtains a proof of the general rule of thumb sometimes quoted in the context of spectral element methods: *π modes per wavelength are needed to resolve a wave.*