Plithogenic Subjective Hyper-Super-Soft Matrices with New Definitions & Local, Global, Universal Subjective Ranking Model

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Plithogenic Subjective Hyper-Super-Soft Matrices with New Definitions & Local, Global, Universal Subjective Ranking Model

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Abstract

In this paper, we initially introduce a novel type of matrix representation of Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Set named as **Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix,** which is generated by multiple parallel sheets of matrices. Furthermore, these parallel sheets are representing parallel universes or parallel realities (a combination of attributes and sub-attributes w.r.t. subjects). We represent cross-sectional cuts of these hyper-soft matrices as parallel sheets (images of the expanded universe). Later, we utilize these Hypersoft matrices to formulate **Plithogenic Subjective Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Super-Soft Matrix.** These matrices are framed by the generalization of Whole Hyper-Soft Set to Subjective Whole Hyper-Soft Set and then their representation in such **hyper-super-soft-matrix (**parallel sheets of matrices) whose elements are matrices. The Hypersoft matrices and hyper-super-soft matrices are tensors of rank three and four, respectively, having three and four indices of variations. Later we provide an application of these Plithogenic Hyper super soft matrices in the form of **Local, Global, Universal Subjective Ranking Model.** The specialty of this model is that it offers precise classification of the universe from micro-universe to macro-universe levels by observing them through several angles of visions in many environments having several ambiguities and hesitation levels. This model provides optimal and neutral values of universes and can compact the expanded universe to a single point in such a way that the compacted universe reflects the cumulative effect of the whole universe. It further offers a transparent ranking by giving a percentage authenticity measure of the ranking. Finally, we provide an application of the model as a numerical example.

Keywords: Plithogenic Hyper-Super-Soft matrices, Sheets of matrices, Expanded Universe, Compacted Universe, Subjective, Local, Global, Universal Ranking,

1.Introduction

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The theory of fuzziness in mathematics was initially introduced by Zadeh [$\mathbf{7}$] in 1965 named as fuzzy set theory (FST). As in crisp Set, a member either completely belongs to a set $\boldsymbol{A}$ or completely doesn't belong to the set $\boldsymbol{A}$ which means we assignee membership to set $\boldsymbol{A}$ either $\mathbf{1}$ or $\mathbf{0}$ while in a fuzzy set, a doubt of belongingness is addressed as a natural trait of the human mind in decision making. The complete membership reduced according to the doubt of belongingness. We may say a fuzzy set is a set where each element of the universe of discourse $\boldsymbol{X}$ has some degree of membership in the unit closed interval $[\mathbf{0}, \mathbf{1}]$ in given set $\boldsymbol{A}$, where $\boldsymbol{A}$ is a subset of universal Set $\boldsymbol{X}$ with respect to an attribute say $\boldsymbol{M}$ with an imposed condition that the sum of membership and non-membership is one unlike crisp Set where the membership is not partial but completely one or completely zero. In fuzzy Set, elements are represented with one variable quantity, i.e., degree of membership noted as $\boldsymbol{\mu_A(x)} \in [\mathbf{0}, \mathbf{1}]\forall \boldsymbol{x} \in \boldsymbol{X}$ and to express the degree of non-membership a notation $\boldsymbol{v_A(x)} \in [\mathbf{0}, \mathbf{1}]\forall \boldsymbol{x} \in \boldsymbol{X}$ was used. $\{\boldsymbol{x}: \boldsymbol{\mu_A(x)}\}$ is the general representation of a fuzzy Set.

Further generalization of fuzzy Set was made by Atanassov [2-3] in 1986, which is known as the Intuitionistic fuzzy set theory (IFS). IFS addresses the doubt on assigning the membership to an element in the previously discussed fuzzy Set. This expanded doubt is the natural trait of the human mind, known as the hesitation factor of IFS. By introducing the hesitation factor in IFS, the sum of opposite membership values was modified. In IFS, the sum of membership, non-membership, and hesitation is one. The degree of hesitation was generally expressed by the notation"$\boldsymbol{\iota}$"now the modified condition is $\boldsymbol{\mu_A(x)} + \boldsymbol{v_A(x)} + \boldsymbol{\iota_A(x)} = \mathbf{1}\forall \boldsymbol{x} \in \boldsymbol{X}$. The elements of IFS expressed by using two variable quantities."$\boldsymbol{\mu_A(x)}$"and "$\boldsymbol{v_A(x)}$"$\{\boldsymbol{x}: (\boldsymbol{\mu_A(x)}, \boldsymbol{v_A(x)})\}$. Later Intuitionistic fuzzy set theory (IFS) was further modernized by Smarandache [1, 15, 17] in 1995, where he assigned an independent degree to the doubt which was aroused in IFS. He represented membership by $\boldsymbol{T(x)}$, the truth of an event and non-membership by $\boldsymbol{F(x)}$ falsity or opposite to truth and the Indeterminacy $\boldsymbol{I(x)}$ is the neutrality between the truth and its opposite. These three factors are considered as independent factors and represented in a unit cube in the non-standard unit interval $]\mathbf{0}^{-}, \mathbf{1}^{+}[$. Smarandache represented the elements of Standard Neutrosophic Set by using three independent quantities i.e.

$\{\boldsymbol{x}: (\boldsymbol{T(x)}, \boldsymbol{I(x)}, \boldsymbol{F(x)})\}$ with condition $\mathbf{0} \leq (\boldsymbol{T(x)} + \boldsymbol{I(x)} + \boldsymbol{F(x)}) \leq \mathbf{3}$

Soft Set was introduced by Molodtsov [5] in 1999, where he exhibited them as a parametrized family of a subset of the universal set. Soft Set further expanded by Deli, Broumi, Çağman [23][24] in 2015.

Later, Smarandache [16] in 2018 generalized the Soft Set to Hypersoft Set and Plithogenic Hypersoft Set by expanding the function of one attribute into a multi attributes/sub-attribute function. He assigned a combined membership $\boldsymbol{\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)}$, non-membership $\boldsymbol{v_{A_1 \times A_2 \times \ldots \times A_N}(x)}$ and indeteriminacy $\boldsymbol{\iota_{A_1 \times A_2 \times \ldots \times A_N}(x)}$ $\forall \boldsymbol{x} \in \boldsymbol{X}$ with condition $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\phi}$ and introduced hybrids of Crisp/ Fuzzy/ Intuitionistic Fuzzy and Neutrosophic hypersoft set and addressed many open problems of development of new literature and MADM techniques.

We have answered some of the open concerns raised by Smarandache [16] in his article "Extension of Soft set to Hypersoft Set and then to Plithogenic Hypersoft Set" published in 2018, in our previous article, titled "Plithogenic Fuzzy Whole Hyper Soft Set, Construction of Local Operators and their Application in Frequency Matrix Multi-

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Attribute Decision Making Technique"[14]. With the help of this Matrix, some local operators for the Plithogenic Fuzzy Hypersoft Set (PFHSS) originated, and their numerical examples constructed. Once these local operators applied to (PFHSS) they gave birth to a new type of Soft Set termed as Plithogenic Fuzzy Whole Hypersoft set (PFWHSS).

Now in this article on its first stage, we represent Plithogenic/Fuzzy/Intuitionistic/Neutrosophic Hyper Soft Set in different and advanced forms of Matrix termed as Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix (PC/F/I/NHS-Matrix). This special type of Matrix is generated by parallel sheets of matrices representing parallel universes or parallel realities. Then by hybridization of this novel Hyper-soft Matrix and Plithogenic Fuzzy Whole Hyper Soft Set, we generate a more generalized and expanded version of Soft Set and name these new Soft Set, "Plithogenic Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper- Soft Set." To represent these sets, we further generate a more expanded form of Hyper-Soft-Matrix naming as Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Super-Soft Matrix (PC/F/I/NHSSM) which is a more advance form of Matrix. These matrices are a hybridization of Super-matrices Introduced by Smarandache [8][9] and Hyper matrices [11].

* Now the question arises why we are using hyper-soft and hyper-super-soft matrices for the expression of Plithogenic Hyper-Soft Set and Plithogenic Subjective Hyper-Soft Set? The answer is very simply and truly convincing that is, the Plithogenic universe is so vast and expanded interiorly (like having Fuzzy, Intuitionistic Fuzzy, Picture Fuzzy, Neutrosophic, Pythogorian, Universes with memberships non-memberships and indeterminacies) and exteriorly (dealing with many attributes, sub-attributes and might be sub-sub-attributes regarding many subjects. The expression for such a vastly expanded universe is not possible by using ordinary algebra or matrices. For the repercussion of an expanded universe with vast indeterminate data, we need some new theories like super or hyper-super algebra or hyper-soft and hyper-super-soft matrices.

* This article is an initial draft or effort for initiating and opening a new dimension of expression by using hyper or hyper-super matrices. It is an expanding field, so in this paper, we introduce names and general expressions of these matrices in many environments. A detailed constructed example is presented in Crisp and Fuzzy environments to keep the length of the article within the required limits of the journal. Later, the detailed constructed examples in other suitable environments would be displayed in the form of upcoming articles. For further motivation see [20-26].

In this paper, we discuss some applications of these matrices in the Crisp and fuzzy environment. These Plithogenic Hyper Soft Matrices and Plithogenic Subjective Hyper Super Soft Matrices are tensors of rank three and four, respectively, having three and four indices of variations. The first index represents rows. The second index represents columns. The third index represents parallel sheets of matrices. The fourth index represents several packets of parallel sheets.

In the second stage, we will utilize these special advanced matrices in the development of a ranking model named "Plithogenic Fuzzy/Intuitionistic/Neutrosophic Local, Global, Universal Subjective Ranking model. This subjective ranking model will provide several types of subjective ranking. Initially, it gives Local Subjective Ranking, then expands it to Global Subjective Ranking than further extends it to Universal Subjective ranking.

This model offers decision making in different levels of fuzziness according to the state of mind of decision-makers. We may vary the level of fuzziness by choosing a suitable environment like Fuzzy, Intuitionistic Neutrosophic, or all environments combined in one environment, etc.

This model offers a transparent decision by analyzing the universe through several angles of visions. The choice of aggregation operator provides a different angle of vision, either optimist pessimist or neutral.

In this model, Local Subjective Ranking is associated with a particular angle of vision. Global Subjective Ranking combines several angles of vision to offer a transparent decision while the Universal, Subjective Ranking is through compacting the expanded universe to have an outer look of the universe.

At the final stage, we provide an application of this subjective ranking model with the help of a numerical example.

We aim to establish such data handling structures that can reduce the complexity and long calculations in decision-making techniques. Decisions will often be made understeer and clear uncertainties (i.e., with incomplete and uncertain knowledge). With the help of our "Subjective ranking model," problems of incomplete and uncertain knowledge of Artificial Intelligence can be solved to a greater extent.

We are giving such a Mathematical structure/pattern that would possibly deal with the expanded data and will shrink its size in such a manner that, in a glimpse, the outcome of it will be obvious to the observer.

2. Preliminaries

In this section, we present some basic definitions of soft sets, fuzzy soft sets, hypersoft Set, crisp hypersoft Set, fuzzy hypersoft Set, plithogenic hypersoft Set, plithogenic crisp hypersoft sets and plithogenic fuzzy hypersoft Set, hyper Matrix and super-matrix.

Definition 2.1 [5] (Soft Set)

"Let $\boldsymbol{U}$ be the initial universe of discourse, and $\boldsymbol{E}$ be a set of parameters or attributes with respect to $\boldsymbol{U}$ let $\boldsymbol{P}(\boldsymbol{U})$ denote the power set of $\boldsymbol{U}$, and $\boldsymbol{A} \subseteq \boldsymbol{E}$ is a set of attributes. Then pair $(\boldsymbol{F}, \boldsymbol{A})$, where $\boldsymbol{F}: \boldsymbol{A} \longrightarrow \boldsymbol{P}(\boldsymbol{U})$ is called Soft Set over $\boldsymbol{U}$. For $\boldsymbol{e} \in \boldsymbol{A}$, $\boldsymbol{F}(\boldsymbol{e})$ may be considered as Set of $\boldsymbol{e}$ elements or $\boldsymbol{e}$ approximate elements $(\boldsymbol{F}, \boldsymbol{A}) = \{(\boldsymbol{F}(\boldsymbol{e}) \in \boldsymbol{P}(\boldsymbol{U}): \boldsymbol{e} \in \boldsymbol{E}, \boldsymbol{F}(\boldsymbol{e}) = \boldsymbol{\varphi}\,\mathbf{if}\,\boldsymbol{e} \notin \boldsymbol{A}\}$ "

Definition 2.2 [6] (Soft Subset)

"For two soft sets $(\boldsymbol{F}, \boldsymbol{A})$ and $(\boldsymbol{G}, \boldsymbol{B})$ over a universe $\boldsymbol{U}$, we say that $(\boldsymbol{F}, \boldsymbol{A})$ is a soft subset of $(\boldsymbol{G}, \boldsymbol{B})$ if (i) $\boldsymbol{A} \subseteq \boldsymbol{B}$, and (ii) $\forall \boldsymbol{e} \in \boldsymbol{A}, \boldsymbol{F}(\boldsymbol{e}) \subseteq \boldsymbol{G}(\boldsymbol{e})$."

Definition 2.3 [7] (Fuzzy Set)

"Let $\boldsymbol{U}$ be the universe. A fuzzy set $\boldsymbol{X}$ over $\boldsymbol{U}$ is a set defined by a membership function $\boldsymbol{\mu_X}$ representing a mapping $\boldsymbol{\mu_X : U \longrightarrow [0,1]}$. The vale of $\boldsymbol{\mu_X}$ $(\boldsymbol{x})$ for the fuzzy set $\boldsymbol{X}$ is called the membership value of the grade of membership of $\boldsymbol{x \in U}$. The membership value represents the degree of belonging to fuzzy set $\boldsymbol{X}$. "A fuzzy set $\boldsymbol{X}$ on $\boldsymbol{U}$ can be presented as follows. $\boldsymbol{X = \{(\mu_X(x)/x): x \in U, \mu_X(x) \in [0,1]\}}$ "

Definition 2.4 [4] (Fuzzy Soft Set)

"Let $\boldsymbol{U}$ be the initial universe of discourse, $\boldsymbol{F(U)}$ be all fuzzy sets over $\boldsymbol{U}$. $\boldsymbol{E}$ be the Set of all parameters or attributes with respect to $\boldsymbol{U}$ and $\boldsymbol{A \subseteq E}$ is a set of attributes. A fuzzy soft set $\boldsymbol{\Gamma_A}$ on the universe $\boldsymbol{U}$ is defined by the Set of ordered pairs as follows, $\boldsymbol{\Gamma_A = \{x, \gamma_A(x): x \in E, \gamma_A(x) \in F(U)\}}$, where $\boldsymbol{\gamma_A: E \longrightarrow F(U)}$ such that $\boldsymbol{\gamma_A(x) = \phi}$ if $\boldsymbol{x \notin A}$, $\boldsymbol{\gamma_A(x) = \left\{\mu_{\gamma_A(x)}(u)/u: u \in U, \mu_{\gamma_A(x)}(u) \in [0,1]\right\}}$ "

Definition 2.5 [16] (Hypersoft set)

"Let $\boldsymbol{U}$ be the initial universe of discourse $\boldsymbol{P(U)}$ the power set of $\boldsymbol{U}$. Let $\boldsymbol{a_1, a_2, \ldots, a_n}$ for $\boldsymbol{n \geq 1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1, A_2, \ldots, A_n}$ with $\boldsymbol{A_i \cap A_j = \varphi}$ for $\boldsymbol{i \neq j}$ and $\boldsymbol{i, j \in \{1, 2, \ldots, n\}}$. Then the pair $(\boldsymbol{F, A_1 \times A \times \ldots \times A_n})$ where, $\boldsymbol{F: A_1 \times A \times \ldots \times A_n \rightarrow P(U)}$, is called a Hypersoft set over $\boldsymbol{U}$"

Definition 2.6 [16] (Crisp Universe of Discourse)

"A Universe of Discourse $\boldsymbol{U_C}$ is called Crisp if $\boldsymbol{\forall x \in U_c}$ $\boldsymbol{x \in 100\%}$ to $\boldsymbol{U_C}$ or membership of $\boldsymbol{x}$ $\boldsymbol{T(x)}$ with respect to $\boldsymbol{A}$ in $\boldsymbol{M}$ is $\boldsymbol{1}$ denoted as $\boldsymbol{x(1)}$"

Definition 2.7 [16] (Fuzzy Universe of Discourse)

"A Universe of Discourse $\boldsymbol{U_F}$ is called Fuzzy if $\boldsymbol{\forall x \in U_C}$ $\boldsymbol{x}$ partially belongs to $\boldsymbol{U_F}$ or membership of $\boldsymbol{x}$ $\boldsymbol{T(x) \subseteq [0,1]}$ where $\boldsymbol{T(x)}$ may be a subset, an interval, a hesitant set, a single value set, denoted as $\boldsymbol{x(T_X)}$"

Definition 2.8 [16] (Crisp Hypersoft set)

"Let $\boldsymbol{U_c}$ be the initial universe of discourse $\boldsymbol{P(U_c)}$ the power set of $\boldsymbol{U}$. let $\boldsymbol{a_1, a_2, \ldots, a_n}$ for $\boldsymbol{n \geq 1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1, A_2, \ldots, A_n}$ with $\boldsymbol{A_i \cap A_j = \varphi}$ for $\boldsymbol{i \neq j}$ and $\boldsymbol{i, j \in \{1, 2, \ldots, n\}}$ Then the pair $(\boldsymbol{F_c,\ A_1 \times A \times \ldots \times A_n})$ where, $\boldsymbol{F_c: A_1 \times A \times \ldots \times A_n \rightarrow P(U_c)}$, is called Crisp Hypersoft set over $\boldsymbol{U_c}$."

Definition 2.9 [16] (Fuzzy Hypersoft set)

"Let $\boldsymbol{U_F}$ be the initial universe of discourse $\boldsymbol{P(U_F)}$ the power set of $\boldsymbol{U_F}$. Let $\boldsymbol{a_1, a_2, \ldots, a_n}$ for $\boldsymbol{n \geq 1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1, A_2, \ldots, A_n}$ with $\boldsymbol{A_i \cap A_j = \varphi}$ for $\boldsymbol{i \neq j}$ and $\boldsymbol{i, j \in \{1, 2, \ldots, n\}}$. Then the pair $(\boldsymbol{F_F, A_1, A_2, \ldots, A_n})$ where, $\boldsymbol{F_F: A_1 \times A \times \ldots \times A_n \rightarrow P(U_F)}$, is called Fuzzy Hypersoft set over $\boldsymbol{U_c}$, Now instead of assigning combined membership $\boldsymbol{\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)}$ $\boldsymbol{\forall x \in X}$ for Hyper Soft sets if each attribute $\boldsymbol{A_j}$ is assigned an individual membership $\boldsymbol{\mu_{A_j}(x)}$, non-membership $\boldsymbol{\upsilon_{A_j}(x)}$ and Indeteriminacy $\boldsymbol{\iota_{A_j}(x) \forall x \in X}$ $\boldsymbol{j = 1, 2, \ldots, n}$ in Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft set then these generalized Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft sets are called Plithogenic Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic Hypersoft sets"

Definition 2.10 [8][9] (Super-matrix)

"A Square or rectangular arrangements of numbers in rows and columns are matrices we shall call them as simple matrices while, super-matrix is one whose elements are themselves matrices with elements that can be either scalars or other matrices. $\boldsymbol{a} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{bmatrix}$, where $\boldsymbol{a_{11}} = \begin{bmatrix} \mathbf{2} & \mathbf{-4} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}, \boldsymbol{a_{12}} = \begin{bmatrix} \mathbf{0} & \mathbf{40} \\ \mathbf{21} & \mathbf{-12} \end{bmatrix}, \boldsymbol{a_{21}} = \begin{bmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{5} & \mathbf{7} \\ \mathbf{-2} & \mathbf{9} \end{bmatrix}$, $\boldsymbol{a_{22}} = \begin{bmatrix} \mathbf{4} & \mathbf{12} \\ \mathbf{-17} & \mathbf{6} \\ \mathbf{3} & \mathbf{7} \end{bmatrix}$ $\boldsymbol{a}$ is a super-matrix".

Note: The elements of super-matrices are called sub-matrices i.e. $\boldsymbol{a_{11}}, \boldsymbol{a_{12}}, \boldsymbol{a_{21}}, \boldsymbol{a_{22}}$ are sub-matrices of the super-matrix $\boldsymbol{a}$.

in this example, the order of super-matrix $\boldsymbol{a}$ is $\mathbf{2} \times \mathbf{2}$ and order of sub-matrices $\boldsymbol{a_{11}}$ is $\mathbf{2} \times \mathbf{2}$, $\boldsymbol{a_{12}}$ is $\mathbf{2} \times \mathbf{2}$ $\boldsymbol{a_{21}}$ is $\mathbf{3} \times \mathbf{2}$ and order of sub-matrix $\boldsymbol{a_{22}}$ is $\mathbf{3} \times \mathbf{2}$, we can see that the order of super-matrix doesn't tell us about the order of its sub-matrices.

Definition 2.11 [11] (Hyper-matrix)

"For $\boldsymbol{n_1}, \ldots, \boldsymbol{n_d} \in \boldsymbol{N}$, a function $\boldsymbol{f}: (\boldsymbol{n_1}) \times \ldots \times (\boldsymbol{n_d}) \rightarrow \boldsymbol{F}$ is a hyper-matrix, also called an order-d hyper-matrix or d-hyper-matrix. We often just write $\boldsymbol{a_{k_1 \ldots k_d}}$ to denote the value $\boldsymbol{f}(\boldsymbol{k_1} \ldots \boldsymbol{k_d})$ of $\boldsymbol{f}$ at $(\boldsymbol{k_1} \ldots \boldsymbol{k_d})$ and think of $\boldsymbol{f}$ (renamed as $\boldsymbol{A}$) as specified by a d-dimensional table of values, writing $\boldsymbol{A} = \left[\boldsymbol{a_{k_1 \ldots k_d}}\right]_{k_1 \ldots k_d}^{n_1, \ldots, n_d}$, A 3-hypermatrix may be conveniently written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices". For example

$$\boldsymbol{A} = \left[\boldsymbol{a_{ijk}}\right] = \begin{bmatrix} \boldsymbol{a_{111}} & \boldsymbol{a_{121}} & \boldsymbol{a_{131}} & \cdot & \boldsymbol{a_{112}} & \boldsymbol{a_{122}} & \boldsymbol{a_{132}} \\ \boldsymbol{a_{211}} & \boldsymbol{a_{221}} & \boldsymbol{a_{231}} & \cdot & \boldsymbol{a_{212}} & \boldsymbol{a_{222}} & \boldsymbol{a_{232}} \\ \boldsymbol{a_{311}} & \boldsymbol{a_{321}} & \boldsymbol{a_{331}} & \cdot & \boldsymbol{a_{312}} & \boldsymbol{a_{322}} & \boldsymbol{a_{332}} \end{bmatrix}$$

3. Plithogenic Hyper-Soft Matrices and Plithogenic Subjective Hyper-Super-Soft Matrices

In this section, we develop some literature about the Plithogenic Hypersoft Set in the following order.

1. Introduction to some basic new concepts and definitions relevant to the hypersoft Set.

2. Generalization of plithogenic whole hypersoft Set to plithogenic subjective hypersoft Set in Crisp, Fuzzy, Intuitionistic, and Neutrosophic environments.

3. Generation of new types of Plithogenic Hypersoft matrix and plithogenic Subjective Hyper-Super-Soft matrix in Crisp, Fuzzy, Intuitionistic, and Neutrosophic environments to represent these new type of sets.

4. Development of expanded and compacted versions of plithogenic Subjective Hyper-Super-Soft Matrix in plithogenic environment.

5. Utilization of new types of matrices for the development of a Subjective ranking model. The specialty of this model is that it offers the classification of non-physical phenomena and Plithogenic

Definition 3.1 (Plithogenic Crisp/ Fuzzy,/Intuitionistic/ Neutrosophic/ Whole Hypersoft set)

If Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hyper Soft set expressed by both individual memberships $\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})$ non-membership $\boldsymbol{\upsilon}_{A_j^k}(\boldsymbol{x_i})$ and Indeterminacy $\boldsymbol{\iota}_{A_j^k}(\boldsymbol{x_i})$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{N}$, $\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{M}$

and $\boldsymbol{k} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{L}$ for each given attribute and Combined memberships $\boldsymbol{\Omega}_{\mathbf{A}_{\alpha}}(\boldsymbol{x}_i)$, non-membership $\boldsymbol{\Phi}_{\mathbf{A}_{\alpha}}(\boldsymbol{x}_i)$ and indeteriminacy $\boldsymbol{\Psi}_{\mathbf{A}_{\alpha}}(\boldsymbol{x}_i)$ $\forall \boldsymbol{x}_i \in \boldsymbol{X}$ for the given $\boldsymbol{\alpha}$ Combination of attributes/sub-attributes and subjects ($\boldsymbol{\alpha}$ universe), then These Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hyper Soft set are called Plithogenic Crisp/Fuzzy/ Intuitionistic Fuzzy/Neutrosophic Whole Hypersoft set.

Definition 3.2 (Plithogenic Crisp/Fuzzy,/Intuitionistic/Neutrosophic Subjective Hypersoft Set)

If Plithogenic Crisp/Fuzzy/Intuitionistic/ Neutrosophic Hypersoft Set is reflected through subjects in such a way that it exhibits both an interior and exterior view of the universe, then, These Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Soft-Set titled, Plithogenic Crisp/Fuzzy/ Intuitionistic/ Neutrosophic Subjective Hypersoft set. The interior view of the universe is reflected through individual memberships $\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)$ non-memberships $\boldsymbol{v}_{A_j^k}(\boldsymbol{x}_i)$ and Indeterminacies $\boldsymbol{\iota}_{A_j^k}(\boldsymbol{x}_i)$ $\forall \boldsymbol{x}_i \in \boldsymbol{X}$ $\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{N}$, $\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{M}$ and $\boldsymbol{k} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{L}$ of subjects and the exterior view is reflected through summative memberships $\boldsymbol{\Omega}_{\mathbf{A}_j}^{\boldsymbol{k}}(\boldsymbol{X})$, non-memberships $\boldsymbol{\Phi}_{\mathbf{A}_j}^{\boldsymbol{k}}(\boldsymbol{X})$ and indeteriminacy $\boldsymbol{\Psi}_{\mathbf{A}_j}^{\boldsymbol{k}}(\boldsymbol{X})$ $\forall \boldsymbol{x}_i \in \boldsymbol{X}$ united specifically for all attribute/sub-attributes and exhibited w.r.t subjects.

Note: In Plithogenic Crisp/Fuzzy/ Intuitionistic Fuzzy/ Neutrosophic Subjective Hypersoft set the united membership $\boldsymbol{\Omega}_{\mathbf{A}_{\alpha}}(\boldsymbol{x}_i)$, non-membership $\boldsymbol{\Phi}_{\mathbf{A}_{\alpha}}(\boldsymbol{x}_i)$ and indeteriminacy $\boldsymbol{\Psi}_{\mathbf{A}_{\alpha}}(\boldsymbol{x}_i)$ $\forall \boldsymbol{x} \in \boldsymbol{X}$ are dependent on individual membership $\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)$, non-membership $\boldsymbol{v}_{A_j^k}(\boldsymbol{x}_i)$ and Indeterminacy $\boldsymbol{\iota}_{A_j^k}(\boldsymbol{x}_i)$.

To achieve a universe, reflected through its subjects (matter for), some terminologies are introduced and described here.

Definition 3.3 (Local Subjective Operators): The aggregation operators, used to cumulate the memberships $\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)$ non-memberships $\boldsymbol{v}_{A_j^k}(\boldsymbol{x}_i)$ and Indeterminacies $\boldsymbol{\iota}_{A_j^k}(\boldsymbol{x}_i)$ Of subjects to achieve a universe reflected through its subjects are termed as local subjective operators.

Definition 3.4 (Local Subjective Ranking): Ranking of subjects under consideration by using a particular aggregation operator is called Local Subjective Ranking. This is the case of the classification of the micro-universe.

Definition 3.5 (Global Subjective Ranking): Ranking of subjects under consideration by using multiple aggregation operators and then obtaining a combined ordering in the form of final ranking is called Global Ranking. This is the case of combining several angles of visions.

Definition 3.6 (Universal Ranking): Ranking of Universes by cumulating effects of attributes and subjects with the help of some local operators is called "Universal Ranking." In Universal Ranking, We converge the whole universe to a single numeric value. Where the combined effect of all attributes and subjects of the universe would be represented by a single numeric value and then writing these converged numeric values in descending order for universal ranking purpose.

Definition 3.7 (Hyper-Soft-Matrix): Let $\boldsymbol{U}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U})$ be the powerset of $\boldsymbol{U}$. Let $\boldsymbol{A}_1^k, \boldsymbol{A}_2^k, \ldots, \boldsymbol{A}_n^k$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, $\boldsymbol{k} = \mathbf{1}, \mathbf{2}, \ldots \boldsymbol{L}$ representing attributes values, a function $\boldsymbol{F}: \boldsymbol{A}_1 \times \boldsymbol{A} \times \ldots \times \boldsymbol{A}_n \rightarrow \boldsymbol{P}(\boldsymbol{U})$ is a hypersoft matrix, also called an order-$\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$ hypersoft matrix or d-hypersoft Matrix ($\boldsymbol{d} = \mathbf{3}$), i.e., a matrix representing a hypersoft set is a hyper-soft matrix (HS-Matrix). As we know, all simple $\boldsymbol{M} \times \boldsymbol{N}$ Matrices on real vector space are tensors of rank 2, so the new Hyper-Soft Matrix having three indices of variations are tensors of rank 3. The Hyper-Soft Matrix is obtained by the generalization of ordinary matrices, known as tensors of rank two. $\boldsymbol{B} = [\boldsymbol{b}_{ijk}]$ is an example of Hyper-Soft Matrix where index $\boldsymbol{i}$ give variation on rows $\boldsymbol{j}$ gives a variation on columns, and $\boldsymbol{k}$ gives variation on clusters of rows and columns.

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The example and detailed illustration of the Hyper-Soft Matrix is presented in the form of the Plithogenic Crisp/Fuzzy Hypersoft Matrix.

The detailed descriptions and applications of the Plithogenic Intuitionistic and Neutrosophic Hyper-Soft Matrix would be exhibited in upcoming versions of research articles.

Definition 3.8 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix): Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Set, when represented in the matrix form are called Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper Soft Matrices, i.e., if $\boldsymbol{B}$ is a hypersoft matrix then $\boldsymbol{B} = [\boldsymbol{b_{ijk}}]$, as $\boldsymbol{b_{ijk}}$ are elements of Matrix and would be expressed in Crisp, Fuzzy, Intuitionistic, Neutrosophic environments with memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-memberships $\boldsymbol{v_{A_j^k}(x_i)}$ and Indeterminacy $\boldsymbol{\iota_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1, 2, \ldots, N}$, $\boldsymbol{i = 1, 2, \ldots, M}$ and $\boldsymbol{k = 1, 2, \ldots, L}$.

A general form of Plithogenic Hyper-Soft Matrix in fuzzy environment is expressed as

$$\boldsymbol{B} = \left[\boldsymbol{\mu_{A_j^k}(x_i)}\right]$$

A general form of Plithogenic Hyper-Soft Matrix in Intuitionistic environment is expressed as

$$\boldsymbol{B} = \left[(\boldsymbol{\mu_{A_j^k}(x_i)}, \boldsymbol{v_{A_j^k}(x_i)}\right]$$

A general form of Plithogenic Hyper-Soft Matrix in Neutrosophic environment is expressed as

$$\boldsymbol{B} = \left[(\boldsymbol{\mu_{A_j^k}(x_i)}, \boldsymbol{\iota_{A_j^k}(x_i)}, \boldsymbol{v_{A_j^k}(x_i)}\right]$$

These matrices described as,

Let $\boldsymbol{U(X)}$ be the universe of discourse in fuzzy or crisp environment and

$$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \ldots \times \boldsymbol{A_N^k} \rightarrow \boldsymbol{P(U)}$$

The Plithogenic Crisp/Fuzzy Hypersoft Set having individual memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1, 2, \ldots, N}$ vary with respect to each attribute $\boldsymbol{A_j}$ and sub-attribute $\boldsymbol{A_j^k}$ ($\boldsymbol{k = 1, 2, 3, \ldots, L}$) where $\boldsymbol{k}$ represent Numeric values of attributes called sub-attributes and $\boldsymbol{i = 1, 2, \ldots, M}$ are the number of subjects under consideration.

In fact in Plithogenic Crisp/Fuzzy Hyper Soft Matrix we have three types of variations introduced on memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$. First variation of $\boldsymbol{i}$ is with respect to subjects representing $\boldsymbol{M}$ rows of $\boldsymbol{M \times N}$ Matrix. The second variation on $\boldsymbol{j}$ is with respect to attributes representing $\boldsymbol{N}$ columns of $\boldsymbol{M \times N}$ Matrix. A third Variation on $\boldsymbol{k}$ is for sub-attributes and represented in the form of $\boldsymbol{L}$ layers or $\boldsymbol{L}$ level sheets of $\boldsymbol{M \times N}$ Matrix. These level cuts of Hyper Matrix are categorized in three types:

1 Vertical front to back and interior level cuts are $\boldsymbol{M \times N}$ Matrix with $\boldsymbol{L}$ level sheets

2 Vertical left to right and interior level cuts are $\boldsymbol{L \times M}$ Matrix with $\boldsymbol{N}$ level sheets

3 Horizontal upper lower and interior level cuts are $\boldsymbol{L \times N}$ Matrix with $\boldsymbol{M}$ level sheets

Definition 3.9 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Cubical Hypersoft Matrix):

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If In Plithogenic Hyper-Soft Matrix $= \boldsymbol{M} = \boldsymbol{N}$. That is, the number of horizontal rows , vertical columns and parallel level cuts are equal, then it is termed a Cubicle Hypersoft Matrix.

Definition 3.10 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Square Rectangular Hypersoft Matrix):

If In plithogenic hypersoft matrix $\boldsymbol{L} \neq \boldsymbol{M} = \boldsymbol{N}$ we shall have an equal number of horizontal rows vertical columns in all sheets, and the number of sheets is different from the number of rows and columns then this hypersoft Matrix is called Square Rectangular Hypersoft Matrices. For example is for the first numeric value of sub-attribute $\boldsymbol{k} = \boldsymbol{1}$ we get the first sheet in the form of Matrix $[\boldsymbol{b}_{ij1}]$ and for second level of sub-attributes, we get the second sheet of Matrix $[\boldsymbol{b}_{ij2}]$. This procedure will continue until the $\boldsymbol{L}$th sheet obtained by taking $\boldsymbol{k} = \boldsymbol{L}$. These all sheets will form a hyper matrix $[\boldsymbol{b}_{ijk}]$.

$\boldsymbol{L}$ −Hypermatrix for $\boldsymbol{L}$ numeric values of attributes forming $\boldsymbol{L}$ sheets of $\boldsymbol{M} \times \boldsymbol{N}$ Matrix, and if we take $\boldsymbol{L} = \boldsymbol{3}$ we will get $\boldsymbol{3}$ −Hypersoft Matrix in the form of three $\boldsymbol{M} \times \boldsymbol{N}$ parallel sheets.

If the environment of the plithogenic hypersoft Set is Crisp/Fuzzy/intuitionistic/neutrosophic, then the plithogenic hypersoft Matrix is called "Plithogenic Crisp/fuzzy/Intuitionistic/Neutrosophic hypersoft matrix."

Plithogenic Crisp/Fuzzy Hypersoft Set when represented in a matrix form having memberships $\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)$, We name it as **Plithogenic Crisp/Fuzzy Hyper-Soft Matrix** is exibited as

Plithogenic Crisp/Fuzzy Hyper-Soft Matrix and level cuts,

$\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)\right]$ by fixing $\boldsymbol{k}$ and giving variation to $\boldsymbol{i}$, $\boldsymbol{j}$ the expanded form of $\boldsymbol{B}$ is obtained as an $\boldsymbol{M} \times \boldsymbol{N}$ matrix, having **L** level cuts. These **L** Level cuts are termed as **Type-1 Level Cuts**.

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & . & . & . & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & . & . & . & \mu_{A_N^1}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & . & . & . & \mu_{A_N^1}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_N^2}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_N^2}(x_M) \end{bmatrix} \\ . \\ . \\ . \\ \begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & . & . & . & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \qquad (1)$$

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Similarly, on the other side, each column of the front sheet with its back columns will form $\boldsymbol{N}$ Level cuts of $\boldsymbol{M} \times \boldsymbol{L}$ matrix termed as **Type-2 Level Cuts**.

These left to right slices are level cuts of type 2, can and be expressed on a two-dimensional page by giving step by step variation to $\boldsymbol{j}$ and expressed as,

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & . & . & . & \mu_{A_1^L}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & . & . & . & \mu_{A_1^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & . & . & . & \mu_{A_1^L}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_2^L}(x_1) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_2^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_2^L}(x_M) \end{bmatrix} \\ . \\ . \\ . \\ \begin{bmatrix} \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & . & . & . & \mu_{A_N^L}(x_1) \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \tag{2}$$

Similarly, **Type-3 Level Cuts** are upper lower and central interior sheets. These sheets form $\boldsymbol{M}$ cuts of $\boldsymbol{N} \times \boldsymbol{L}$ matrix.

These top to bottom slices are level sheets of type 3 and can be expressed on a two-dimensional page by giving step by step variation to $\boldsymbol{i}$ in $\boldsymbol{B} = \left[\mu_{A_j^k}(x_i)\right]$ and described as,

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$$B = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & . & . & . & \mu_{A_1^L}(x_1) \\ \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_2^L}(x_1) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & . & . & . & \mu_{A_N^L}(x_1) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & . & . & . & \mu_{A_1^L}(x_2) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_2^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \end{bmatrix} \\ . \\ . \\ . \\ \begin{bmatrix} \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & . & . & . & \mu_{A_1^L}(x_M) \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_2^L}(x_M) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \quad (3)$$

In $\boldsymbol{\mu_{A_j^k}(x_i)}$ $\boldsymbol{i}$ provides row-wise variations of subjects, $\boldsymbol{j}$ provides column-wise variations of attributes, and $\boldsymbol{k}$ provides variations of sub-attributes in sheets of matrices. These are hyper matrices $[\boldsymbol{a_{ijk}}]$ In the journal of order $\boldsymbol{M \times N \times L}$.

Definition 3.11 (Hyper-Super Matrix): Such a Hyper matrix whose elements are presented in matrix form is titled as Hyper-Super Matrix. These matrices are expressed by using more than two variation indices. These Hyper Super matrices have multiple sheets of ordinary matrices, i.e., $= \begin{bmatrix} [\boldsymbol{a_{ijk}}][\boldsymbol{b_{ijk}}] \\ [\boldsymbol{c_{ijk}}][\boldsymbol{d_{ijk}}] \end{bmatrix}$, B is an example of a hyper-super matrix. The elements of this Matrix ($[\boldsymbol{a_{ijk}}]$, $[\boldsymbol{b_{ijk}}]$, $[\boldsymbol{c_{ijk}}]$ and $[\boldsymbol{d_{ijk}}]$) are hyper-matrices.

* Tthe literature regarding to Hyper super matrices like operators properties application would be explored in upcoming articles)

A Hyper-Super Matrix, used to express a Plithogenic Subjective Hypersoft Set, is an example of Hyper-Super Matrix. The elements of this Matrix are matrices. These hyper-super matrices have multiple packets of parallel sheets and have more than three variations ($\boldsymbol{d > 3}$) .

The examples of HSS-Matrix are **Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Super-Soft matrices**

A detailed illustration of the Plithogenic Crisp/fuzzy Hyper-Super-Soft Matrix is presented here, and the rest of the introduced matrices would be presented in upcoming articles.

3.12 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective-Hyper-Super-Soft Matrices):

If the matrix representation form of PC/FWHSS is in such a way that the last column matrix of each sheet of the hyper Matrix represents the Combined effect of memberships for sub-attributes with respect to each subject under consideration, then these matrices are called Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Super-Soft Matrices whose last column of cumulative memberships makes another matrix, we can see here the elements of hyper matrices are some matrices, so these matrices are basically hyper super matrices because these are such sheets of matrices whose elements are matrices. For example, if the combined memberships for Plithogenic Crisp/ Fuzzy Subjective Hyper Soft Set are denoted by $\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})$ for $\boldsymbol{x_i}$ subject, combined attributes for $\boldsymbol{k}$th level of sub-attributes denoted by $\mathbf{A}^{\boldsymbol{k}}$ and operator used to cumulate memberships for given attributes denoted by $\boldsymbol{t}$, ($\boldsymbol{t} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$) one of the four local previously constructed operators (ref), then $\boldsymbol{B}_{\mathbf{S}_t} = \left[[\boldsymbol{b_{ijk}}][\boldsymbol{b_{ikt}}]\right]$ Is Plithogenic Crisp/ Fuzzy Subjective Hyper super Soft Matrix, which will provide categorization of subjects from last $\boldsymbol{col}$ memberships values. As $\boldsymbol{b_{ijk}}$ the elements of Matrix representing crisp/ fuzzy memberships and $\boldsymbol{b_{ikt}}$ representing cumulative crisp/fuzzy memberships for some subject with respect to all given attributes can be expressed as $\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})$.

More generalized form of $\boldsymbol{B}_{\mathbf{S}_t}$ is. $\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ and further expanded form with respect to $\boldsymbol{i}$ and $\boldsymbol{j}$ is,

$$\boldsymbol{B}_{\mathbf{S}_t} = \left[\begin{bmatrix} \boldsymbol{\mu}_{A_1^k}(\boldsymbol{x_1}) & \boldsymbol{\mu}_{A_2^k}(\boldsymbol{x_1}) & . & . & . & \boldsymbol{\mu}_{A_N^k}(\boldsymbol{x_1}) \\ \boldsymbol{\mu}_{A_1^k}(\boldsymbol{x_2}) & \boldsymbol{\mu}_{A_2^k}(\boldsymbol{x_2}) & . & . & . & \boldsymbol{\mu}_{A_N^k}(\boldsymbol{x_2}) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \boldsymbol{\mu}_{A_1^k}(\boldsymbol{x_M}) & \boldsymbol{\mu}_{A_2^k}(\boldsymbol{x_M}) & . & . & . & \boldsymbol{\mu}_{A_N^k}(\boldsymbol{x_M}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_1}) \\ \boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_2}) \\ . \\ . \\ . \\ \boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_M}) \end{bmatrix}\right] \quad (4)$$

In Plithogenic Crisp/ Fuzzy Subjective Hyper super Soft Matrix, $\boldsymbol{B}_{\mathbf{S}_t}$ we give four type of variations

Example: Different cross-sectional cuts of $\boldsymbol{B}_{\mathbf{S}_t}$ a super hyper matrix will form packets of multiple parallel sheets, i.e., multiple combinations of sheets, which is a more expanded universe.

In $\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$, Variations of $\boldsymbol{i}$ generates rows, variations of $\boldsymbol{j}$ generates columns, variations of $\boldsymbol{k}$ generates combinations of rows and columns as parallel layers of $\boldsymbol{M} \times \boldsymbol{N}$ matrix and variation of $\boldsymbol{t} = \mathbf{1}, \mathbf{2}, \ldots \boldsymbol{P}$ will provide $\boldsymbol{P}$ set of multiple $\boldsymbol{L}$ parallel layers of matrices.

These sets of combinations of sheets may represent parallel universes, on a two-dimensional sheet of paper the expanded form of general hyper super-matrix

$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ can be represented as

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$$
\begin{bmatrix}
\left[\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^1_{A^1}(x_1) \\ \Omega^t_{A^1}(x_2) \\ \vdots \\ \Omega^1_{A^1}(x_M) \end{bmatrix}\right] \\
\left[\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^1_{A^2}(x_1) \\ \Omega^1_{A^2}(x_2) \\ \vdots \\ \Omega^1_{A^2}(x_M) \end{bmatrix}\right] \\
\vdots \\
\left[\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^1_{A^L}(x_1) \\ \Omega^1_{A^L}(x_2) \\ \vdots \\ \Omega^1_{A^L}(x_M) \end{bmatrix}\right] \\
\left[\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^2_{A^1}(x_1) \\ \Omega^2_{A^1}(x_2) \\ \vdots \\ \Omega^2_{A^1}(x_M) \end{bmatrix}\right] \\
\left[\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^2_{A^2}(x_1) \\ \Omega^2_{A^2}(x_2) \\ \vdots \\ \Omega^2_{A^2}(x_M) \end{bmatrix}\right] \\
\vdots \\
\left[\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^2_{A^L}(x_1) \\ \Omega^2_{A^L}(x_2) \\ \vdots \\ \Omega^2_{A^L}(x_M) \end{bmatrix}\right] \\
\vdots \\
\left[\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^P_{A^1}(x_1) \\ \Omega^P_{A^1}(x_2) \\ \vdots \\ \Omega^P_{A^1}(x_M) \end{bmatrix}\right] \\
\left[\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^P_{A^2}(x_1) \\ \Omega^P_{A^2}(x_2) \\ \vdots \\ \Omega^P_{A^2}(x_M) \end{bmatrix}\right] \\
\vdots \\
\left[\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^P_{A^L}(x_1) \\ \Omega^P_{A^L}(x_2) \\ \vdots \\ \Omega^P_{A^L}(x_M) \end{bmatrix}\right]
\end{bmatrix}
\tag{5}
$$

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It is observed that the multiple sets of parallel sheets (universes) are achieved by giving step by step variation to the fourth index $\boldsymbol{t}$, which is used to represent several aggregation operators to homogenize attributes established as local operators.

It is interesting to note That if we see the hyper-super-soft-matrix on a two-dimensional screen, then columns of sheets of $\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$ Matrix will form a general $\boldsymbol{M} \times \boldsymbol{L}$ matrices.

Greater the numeric value of $\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})$ better is the $\boldsymbol{x_i}$ subject under consideration. This shows that the whole effect of the Matrix can be visualized by observing the last column. $\left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ is representing a Hyper Super Soft Matrix and for subjective categorization with respect to different levels of sub-attributes is obtained by keeping in view a single sheet at a time, to achieve the purpose we shall use one of the sheets of this hyper super-matrix $\left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ by fixing $\boldsymbol{k}$ for a specific level of the attribute, and then we will vary " $\boldsymbol{t}$ " which is used to represent a particular aggregation operator, and this operator could be used to cumulate attributes effect at a certain level.

$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^1}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\mathbf{1}}_{\mathbf{A}^\alpha}(\boldsymbol{x_i})\right]\right]$ will represent a single sheet of hyper super-matrix for $\boldsymbol{\alpha}$ universe at a fixed numeric value of attribute of first level $\boldsymbol{k} = \mathbf{1}$ and using particular operator $\boldsymbol{t} = \mathbf{1}$.

By fixing a combination of attributes, sub-attributes and subjects means considering one of the parallel sheets (one parallel universe) of the hyper-super-soft- Matrix by using Plithogenic Fuzzy Subjective Hyper Super Soft Matrices.

One may decide which subject is superior with respect to all attributes by analyzing the last column of the sheet (case of local Ranking), the subject which corresponds to the greater numeric value in the last column will be considered better, and then we will write them in descending order.

4. Application

Local, Global and Universal Subjective Ranking Model

In this section, we utilize the local operators constructed in an earlier published article ([26]) in formulation of Subjective Ranking model in plithogenic Crisp/Fuzzy environment.

1. The specialty of this model is that it offers the classification of subjects by observing them through several angles of vision.

2. Different angles of visions can be expressed by choosing a suitable local operator

3. This model offers classification in many environments where every environment has its ambiguity and uncertainty level.

4. This Subjective ranking model provides the ranking from micro-universe to the macro universe level.

5. Initially, this model provides the internal Ranking named "Local Subjective ranking" (classification of subjects of micro-universe). In Local Ranking, one perticular local operators is used for the classification purpose.

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6. At the next stage, this model offers an external classification of subjects named "Global Subjective Ranking." In Global Ranking, we combine several operators to formulate an expression that reflected through many angles of visions.

7. This model offers the third type of subjective Ranking named "Universal Subjective Ranking (case of macro universe Classification)

8. This model would provide extreme and neutral values of universes that would be helpful to find out the optimal and neutral behaviors of all types of universe micro to macro levels.

9. At the last stage, it provides a measure of the authority of classification by using the frequency matrix.

Initially, we consider the case of plithogenic Crisp/fuzzy Subjective-Hyper-Super-Soft matrices for classification of subjects. Later we may generalize these models in Plithogenic Intuitionistic and Plithogenic Neutrosophic environments. Describe the subjective ranking model in the given steps.

Step 1. Construction of Universe: Consider universe of discourse $\boldsymbol{U} = \{\boldsymbol{x_i}\}$ $\boldsymbol{i = 1, 2, 3, \ldots, P}$. Consider some attributes and subjects needed to be classified where attributes are $\boldsymbol{A_j^k}$ $\boldsymbol{j = 1, 2, 3, \ldots, N}$ and $\boldsymbol{k = 1, 2, \ldots, L}$ represents numeric values (levels) of $\boldsymbol{A_j}$, and subjects are $\boldsymbol{T = \{x_i\} \subset U}$ where $\boldsymbol{i}$ can take some values from $\boldsymbol{1}$ to $\boldsymbol{M}$ such that $\boldsymbol{M \leq p}$ and define mappings $\boldsymbol{F}$ and $\boldsymbol{G}$ such that,

$\boldsymbol{F: A_1^k \times A_2^k \times A_3^k \times \ldots \times A_N^k \to P(U)}$ For some fixed $\boldsymbol{k}$ (level$\boldsymbol{1}$)

$\boldsymbol{G: A_1^k \times A_2^k \times A_3^k \times \ldots \times A_N^k \to P(U)}$ For some different fixed $\boldsymbol{k}$ (level $\boldsymbol{2}$)

Step 2. Plithogenic Crisp/Fuzzy Hyper Soft Matrix: Write the data or information (Memberships) in the form of Plithogenic Fuzzy Hyper Soft Matrix $\boldsymbol{B = \left[\mu_{A_j^k}(x_i)\right]}$

Step 3. Plithogenic Crisp/Fuzzy Subjective Hyper-super-Soft Matrix: By using previously constructed local operators formulate Plithogenic Fuzzy Hyper super Soft Matrix

$$\boldsymbol{B_{S_t} = \left[\left[\mu_{A_j^k}(x_i)\right]\left[\Omega^t_{\mathrm{A}^k}(x_i)\right]\right]}.$$

In formulating $\boldsymbol{\left[\Omega^t_{\mathrm{A}^k}(x_i)\right]}$ we might be handling some favorable, some neutral and some non-favorable attributes, the non-favorable attributes can be handled by replacing their corresponding memberships with standard compliments i.e. $\boldsymbol{\mu^c_{A_j^k}(x_i) = 1 - \mu_{A_j^k}(x_i)}$, While for neutral attributes, the regular memberships can be used.The cumulative memberships $\boldsymbol{\Omega^t_{\mathrm{A}^k}(x_i)}$ are obtained by uniting regular memberships for favorable and neutral attributes and compliments of memberships for non-favorable attributes by using local operators.

Step 4. Local Ranking: The Local Ranking would be obtained by observing cumulative memberships $\boldsymbol{\Omega^t_{\mathrm{A}^\alpha}(x_i)}$ of last column of

$$\boldsymbol{B_{S_t} = \left[\left[\mu_{A_j^k}(x_i)\right]\left[\Omega^t_{\mathrm{A}^k}(x_i)\right]\right]},$$

Higher the membership value better is the subject corresponding to that membership.

At this stage, the classification of all sheets or one selected sheet would be provided according to the requirement. The process can be stopped here if transparent Ranking is achieved.

If there are some ties or ambiguities, would be removed in the next step, and more transparent Ranking can be provided with authenticity measurement.

Step 5. Global Ranking: Final Global Subjective ordering can be provided by using the Frequency matrix.

$$F_{ij} = \begin{array}{c} x_1 \\ x_2 \\ . \\ . \\ . \\ x_M \end{array} \begin{bmatrix} f_{11} & f_{12} & . & . & . & f_{1M} \\ f_{21} & f_{22} & . & . & . & f_{2M} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ f_{M1} & f_{M2} & . & . & . & f_{MM} \end{bmatrix} \tag{6}$$

In F_{ij} the elements of the first column will represent the frequency of obtaining the first position of given subjects. The elements of the second column will represent the frequency of obtaining the second position, and so on. To find out which subject would attain the first position we observe the entries of the first column of F_{ij} the subject corresponding to the highest value of the first column attains the first position, and then we delete the first position and the subject who have achieved that position. Afterward, for the selection of the second position, add the remaining frequencies of first positions in the frequencies of the second column and then look for the highest frequency in that column.

Once second position is decided to delete column and row of that position correspondingly and continue the procedure until the latest position is assigned

Step 6. Authenticity Measure of Global Ranking: We may check the authenticity in the last step by taking ratios.

percentage authenticity of j $_{th}$ position for i $_{th}$ subject

$$= \frac{\text{Highest frequency of } j_{th} \text{ position}}{\text{Total frequency of } j_{th} \text{position}} \times 100 = \frac{\max_i (f_{ij})}{\sum_i (f_{ij})} \times 100 \tag{7}$$

Step 7. Universal Ranking: If comparison of universes is required we may provide by combining cumulative memberships of last column for fixed k and t for each universe. The universe having the largest cumulative memberships is considered as a better universe and then writes these cumulative memberships in a descending order.

$$\Omega_{k1} = \max_i \left[\Omega_{A^k}(x_i)\right] \tag{8}$$

$$\Omega_{k2} = \max_i \left[\Omega^2_{A^k}(x_i)\right] \tag{9}$$

$$\Omega_{k3} = \frac{\sum_i \left[\Omega^3_{A^k}(x_i)\right]}{n} \tag{10}$$

5 Local operators for Construction of Plithogenic crisp/fuzzy Subjective Hyper Super Soft matrices.

These local operators for the plithogenic Crisp/Fuzzy hypersoft Set can be utilized to formulate Plithogenic Crisp/Fuzzy Subjective Hyper Super Soft Matrices.

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By using local disjunction, local conjunction and local averaging operators we developed a combined (whole) memberships $\boldsymbol{\Omega}^{t}_{\mathrm{A}^{\alpha}}(\boldsymbol{x_i})$ for Plithogenic Hyper-Soft-Set. By applying local aggregation operators on Crisp/fuzzy Subjective hyper soft matrices $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]$ the last column of cumulative memberships $\boldsymbol{\Omega}^{t}_{\mathrm{A}}(\boldsymbol{x_i})$ is obtained. To achieve the purpose we use three local operators, $\boldsymbol{t = 1}$ is used for $\mathbf{max}$, $\boldsymbol{t = 2}$ is used for $\mathbf{min}$, and $\boldsymbol{t = 3}$ is used for averaging operator described as,

$$\cup_i\left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Max}\left(\mu_{A_j^k}(x_i)\right) = \Omega^1_{\mathrm{A}}(x_i) \qquad (12)$$

$$\cap_i\left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Min}(\mu_{A_j^k}(x_i)) = \Omega^2_{\mathrm{A}}(x_i) \qquad (13)$$

$$\Gamma_i\left(\mu_{A_j^k}(x_i)\right) = \frac{\sum_j\left(\mu_{A_j^k}(x_i)\right)}{M} = \Omega^3_{\mathrm{A}}(x_i) \qquad (14)$$

6 Application of Subjective ranking model in Crisp environment

Numerical Example:

To achieve the purpose, we will develop plithogenic Fuzzy hyper-soft matrix $\boldsymbol{B}$ for two Plithogenic crisp Hyper Soft Sets with $\boldsymbol{\alpha}$ Combination and $\boldsymbol{\beta}$ Combination of attributes, i.e., for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes.

By choosing different numeric values of $\boldsymbol{k}$ consider $\boldsymbol{\alpha}$ Combination of attributes for $\boldsymbol{F}$ and $\boldsymbol{\beta}$ Combination of attributes for $\boldsymbol{G}$.

Step 1. Construction of Universe: Consider $\boldsymbol{U} =$ is a set of five candidates in the Mathematics department. $\boldsymbol{T} = \{\mathbf{Peter}, \mathbf{Aina}, \mathbf{kitty}\} \subset \boldsymbol{U}$ is the Set of three candidates, $\boldsymbol{T} = \{\mathbf{Peter}, \mathbf{Aina}, \mathbf{kitty}\}$Who have Applied for the selection of the post of Assistant professor are our subjects required to be classified with respect to following $\boldsymbol{A_j^k}$ attributes and sub-attributes.

$\boldsymbol{A_1^k} =$ Subject skill area with numeric values, $\boldsymbol{k = 1, 2}$

$\boldsymbol{A_1^1} =$ Pure Mathematics, $\boldsymbol{A_1^2} =$ Applied Mathematics

$\boldsymbol{A_2^k} =$ Qualification with numeric values, $\boldsymbol{k = 1, 2}$

$\boldsymbol{A_2^1} =$ Qualification like MS or Equivalent, $\boldsymbol{A_2^2} =$ Higher Qualification like Ph.D. or Equivalent

$\boldsymbol{A_3^k} =$ Teaching experience with numeric values, $\boldsymbol{k = 1, 2}$

$\boldsymbol{A_3^1} =$ Five years or less, $\boldsymbol{A_3^2} =$ More than Five years

$\boldsymbol{A_4^k} =$ Age , with numeric values $\boldsymbol{k = 1, 2}$

$\boldsymbol{A_4^1} =$ Age is less than forty years $\boldsymbol{A_4^2} =$ Age is more than forty years

Consider mapping $\boldsymbol{F}$ and $\boldsymbol{G}$ such that,

$\boldsymbol{F}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \ldots \times \boldsymbol{A_N^k} \to \boldsymbol{P(U)}$ (choosing some numeric values of $\boldsymbol{A_j^k}$ $\boldsymbol{k} \in (\boldsymbol{1}, \boldsymbol{L})$

$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \ldots \times \boldsymbol{A_N^k} \rightarrow \boldsymbol{P(U)}$ (choosing some different numeric values of $\boldsymbol{A_j^k}$ $\boldsymbol{k} \in (\boldsymbol{1}, \boldsymbol{L})$Let the these candidates are considered as subjects under consideration, and attributes are $\boldsymbol{A_j^k}$ $\boldsymbol{j = 1, 2, 3, 4}$ and $\boldsymbol{k = 1, 2, 3}$

we are looking for a single employ amongst the three candidates, $\boldsymbol{T} = \{\textbf{Peter}, \textbf{Aina}, \textbf{kitty}\} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$, where $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}$ represent $\boldsymbol{x_i}$ subjects under consideration, initially we consider the first level for $\boldsymbol{k = 1}$ i.e.

1. Subject skill area: Pure Mathematics $\boldsymbol{j = 1}, \boldsymbol{k = 1}$

2. Qualification: Qualification like MS or Equivalent $\boldsymbol{j = 2}, \boldsymbol{k = 1}$

3. Teaching experience: Five years or less $\boldsymbol{j = 3}, \boldsymbol{k = 1}$

4. Age: Age required is less than forty years $\boldsymbol{j = 4}, \boldsymbol{k = 1}$

Let the Function is $\boldsymbol{F}$ is defined as,

$\boldsymbol{F}(\boldsymbol{A_1^1}, \boldsymbol{A_2^1}, \boldsymbol{A_3^1}, \boldsymbol{A_4^1}) = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ let we name $\boldsymbol{A_1^1}$, $\boldsymbol{A_2^1}$, $\boldsymbol{A_3^1}$, $\boldsymbol{A_4^1}$ as $\boldsymbol{\alpha}$ Combination representing the first level for $\boldsymbol{k = 1}$

Let the Function is $\boldsymbol{G}$ is defined as,

$$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \boldsymbol{A_4^k} \rightarrow \boldsymbol{P(U)}$$

We are looking for another employ amongst these three candidates for the Category of applied mathematics with a different combination of attributes say $\boldsymbol{\beta}$ Combination for next level ($\boldsymbol{k = 2}$).

1. Subject skill area: Applied Mathematics $\boldsymbol{j = 1}, \boldsymbol{k = 2}$

2. Qualification: Higher Qualification like Ph.D. or equivalent $\boldsymbol{j = 2}, \boldsymbol{k = 2}$

3. Teaching experience: More than five years $\boldsymbol{j = 3}, \boldsymbol{k = 2}$

4. Age: Age is more than forty years $\boldsymbol{j = 3}, \boldsymbol{k = 2}$

$\boldsymbol{G}(\boldsymbol{A_1^2}, \boldsymbol{A_2^2}, \boldsymbol{A_3^2}, \boldsymbol{A_4^2}) = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ let we name $\boldsymbol{A_1^2}, \boldsymbol{A_2^2}, \boldsymbol{A_3^2}, \boldsymbol{A_4^2}$ as $\boldsymbol{\beta}$ Combination.with respect to. $\boldsymbol{T} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ Have memberships In PCHSS, PFHSS, which are assigned by decision-makers in crisp environment memberships, are considered as $\mathbf{1}$, or $\mathbf{0}$.

While in a fuzzy environment, memberships are assigned by decision-makers between $\mathbf{0}$ and $\mathbf{1}$ by using five-point scale linguistic chart.

Step 2. Plithogenic Crisp Hyper Soft Matrix:

With respect to $\boldsymbol{T} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ memberships In PCHSS are,

$$\boldsymbol{F}(\boldsymbol{A_1^1}, \boldsymbol{A_2^1}, \boldsymbol{A_3^1}, \boldsymbol{A_4^1}) = \boldsymbol{F}(\boldsymbol{\alpha}) = \{\boldsymbol{x_1}(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \boldsymbol{x_2}(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \boldsymbol{x_3}(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})\}$$

$$\boldsymbol{G}(\boldsymbol{A_1^2}, \boldsymbol{A_2^2}, \boldsymbol{A_3^2}, \boldsymbol{A_4^2}) = \boldsymbol{F}(\boldsymbol{\beta}) = \{\boldsymbol{x_1}(\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}), \boldsymbol{x_2}(\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}), \boldsymbol{x_3}(\mathbf{0}, \mathbf{0}, \mathbf{0},)\}$$

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The **Plithogenic Crisp Hyper Soft matrix** $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)\right]$ $\mathbf{1}$for $\boldsymbol{k} = \mathbf{1}, \boldsymbol{j} = \mathbf{1,2,3,4}$ and $\boldsymbol{i} = \mathbf{1,2,3}$ i.e. $\boldsymbol{\alpha}$ Combination (first level sheet) and $\boldsymbol{k} = \mathbf{2}, \boldsymbol{j} = \mathbf{1,2,3,4}$ and $\boldsymbol{i} = \mathbf{1,2,3}$ i.e.$\boldsymbol{\beta}$ Combination (second level sheet) will be represented as $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)\right]$

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \tag{14}$$

In Crisp Hyper Soft matrix $\boldsymbol{B}$ the first level sheet for $\boldsymbol{\alpha}$ Combination of attribute values all memberships are one means the members fulfill the first level requirements fully but for second-level sheet i.e.$\boldsymbol{\beta}$ combination members may fulfill the requirements partially, but the crisp universe don't deal with partial belongingness, so all memberships are zero.

Step 3. Plithogenic Crisp Subjective Hyper-Super-Soft Matrix:

The Plithogenic Crisp Hyper Soft matrix $\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x}_i)\right]\right]$ by using eq. $\mathbf{5}$ for $= \mathbf{1,2,3}$, $\boldsymbol{j} = \mathbf{1,2,3,4}$ and $\boldsymbol{k} = \mathbf{1}$, i.e., .$\boldsymbol{\alpha}$ Combination (first level sheet), $\boldsymbol{k} = \mathbf{2}$, $\boldsymbol{\beta}$ Combination (second level sheet) $\boldsymbol{t} = \mathbf{1}$ (disjunction operator) first set of two-level sheets, $\boldsymbol{t} = \mathbf{2}$ (conjunction operator) second Set of two-level sheets will be represented as,

$$\boldsymbol{B}_{\mathbf{S}_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{15}$$

It is obvious from this HSS-Matrix that in Crisp environment $\boldsymbol{B}_{\boldsymbol{St}}$ representing trivial cases where the subjective categorization is not visible because all values in the last column are the same, so we in future Classifications we will use Plithogenic Fuzzy subjective Hyper-soft matrices for subjective categorization. Further, it is observed that the crisp universe case is trivial, where the categorization is not possible. So next steps would not proceed.

Note: In the next levels, we will consider only a fuzzy environment to achieve transparent classification.

Application of Subjective ranking model in Fuzzy environment

Step1. Construction of Universe: To formulate Plithogenic Fuzzy Hyper Soft Matrix Step 1 and Step 2 are the same as already described in the crisp universe case.

Step 2. Plithogenic Fuzzy Hyper-Super-Soft Matrix:

Memberships of $\boldsymbol{T} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3\}$ In PFHSS with respect to given attribute memberships are assigned by the decision-makers by using linguistic five point scale method,[1], [18], [34] [35]

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$$F(A_1^1, A_2^1, A_3^1, A_4^1) = F(\alpha) = \{x_1(0.3, 0.6, 0.4, 0.5), x_2(0.4, 0.5, 0.3, 0.1), x_3(0.6, 0.3, 0.3, 0.7)\}$$

$$G(A_1^2, A_2^2, A_3^2, A_4^2) = G(\beta) = \{x_1(0.5, 0.3, 0.2, 0.6), x_2(0.6, 0.7, 0.8, 0.4), x_3(0.7, 0.7, 0.5, 0.9)\}$$

The Plithogenic Fuzzy Hyper Soft matrix $B = \left[\mu_{A_j^k}(x_i)\right]$ by using eq.1 for $i = 1, 2, 3$,

$j = 1, 2, 3, 4$ and $k = 1$, i.e.α Combination (first level sheet) , $k = 2$, β Combination (second level sheet) will be represented as,

$$B = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \end{bmatrix} \tag{16}$$

Step 3. Plithogenic Fuzzy Subjective Hyper Super Soft Matrix:

The Plithogenic Fuzzy Hyper Super Soft matrix $B_{S_t} = \left[\left[\mu_{A_j^k}(x_i)\right]\left[\Omega_{A^k}^t(x_i)\right]\right]$ by using eq. 5, 11, 12, and 13 for $i = 1, 2, 3$, $j = 1, 2, 3, 4$ and $k = 1$, i.e., α combination (first level sheet), $k = 2$, β combination (second level sheet), $t = 1$, (disjunction operator) first set of two-level sheets, $t = 2$, (conjunction operator) second set of two-level sheets, $t = 3$ (averaging operator) a third set of two-level sheets will be represented as,

$$B_{S_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.5 \\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.8 \\ 0.9 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.1 \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.4 \\ 0.5 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.45 \\ 0.35 \\ 0.475 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.625 \\ 0.7 \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{17}$$

Step 4. Local Subjective Ranking:

$B_{S_{1\alpha}}$ In eq. 17 provides the local ordering of subjects for α Combination of attributes or α universe (first level sheet for $k = 1$) by using the first operator ($t = 1$) given in eq. 11 as

$$x_3 \succ x_1 \succ x_2 \tag{18}$$

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$\boldsymbol{B}_{\boldsymbol{S}_{2\alpha}}$in eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes or $\boldsymbol{\alpha}$ universe (first level sheet for $\boldsymbol{k}=\mathbf{1}$) by using second operator ($\boldsymbol{t}=\mathbf{2}$) given in eq. $\mathbf{12}$ as

$$\boldsymbol{x_3} = \boldsymbol{x_1} \succ \boldsymbol{x_2} \qquad (19)$$

which shows a tie between $\boldsymbol{x_1}$ and $\boldsymbol{x_3}$ which can be removed in the final Global Ranking of subjects by using frequency matrix $\boldsymbol{F_{ij}}$

$\boldsymbol{B}_{\boldsymbol{S}_{3\alpha}}$ In eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes ($\boldsymbol{\alpha}$ universe) by using a third operator ($\boldsymbol{t}=\mathbf{3}$) given in eq. 13 as

$$\boldsymbol{x_3} \succ \boldsymbol{x_1} \succ \boldsymbol{x_2} \qquad (20)$$

Similarly

$\boldsymbol{B}_{\boldsymbol{S}_{1\beta}}$ in eq. $\mathbf{17}$ provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using first operator ($\boldsymbol{t}=\mathbf{1}$) given in eq. 11 as

$$\boldsymbol{x_3} \succ \boldsymbol{x_2} \succ \boldsymbol{x_1} \qquad (21)$$

$\boldsymbol{B}_{\boldsymbol{S}_{2\beta}}$ in eq.. 17provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using the second operator ($\boldsymbol{t}=\mathbf{2}$) given in eq. 12as

$$\boldsymbol{x_3} \succ \boldsymbol{x_2} \succ \boldsymbol{x_1} \qquad (22)$$

$\boldsymbol{B}_{\boldsymbol{S}_{3\beta}}$ in eq.. 17 provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using the third operator ($\boldsymbol{t}=\mathbf{3}$) given in eq. 13 as

$$\boldsymbol{x_3} \succ \boldsymbol{x_2} \succ \boldsymbol{x_1} \qquad (23)$$

We can observe that the three local ordering of subjects under consideration in $\boldsymbol{\alpha}$ Combination can be provided by using three operators described in eq., 11, 13, 12, to find cumulative memberships for $\boldsymbol{\alpha}$ Combination first, then for $\boldsymbol{\beta}$ Combination and then writing in descending order.

Step 5. Global Subjective Ranking :

Final global ordering of subjects can be provided by using frequency matrix $\boldsymbol{F_{ij}}$ described in eq.. 6, $\mathbf{18}$, 19, and 20

In the frequency matrix $\boldsymbol{F_{ij}^{\alpha}}$ which is a square matrix of frequencies positions for first level sheet $\boldsymbol{\alpha}$ Combination the $\boldsymbol{colF_{ij}^{\alpha}}$ represents frequencies of

positions, i.e., the entries of the first column represents the frequencies of attaining first position by given subjects while $\boldsymbol{rowF_{ij}}$ represents subjects.

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$$\boldsymbol{F_{ij}^{\alpha}} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Global Ranking of subjects from $\boldsymbol{F_{ij}^{\alpha}}$ is $\boldsymbol{x_3 \succ x_1 \succ x_2}$ (24)

The frequency matrix for $\boldsymbol{\beta}$ universe is $\boldsymbol{F_{ij}^{\beta}}$

$$\boldsymbol{F_{ij}^{\beta}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{3} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Global Ranking of subjects from $\boldsymbol{F_{ij}^{\beta}}$ is $\boldsymbol{x_3 \succ x_2 \succ x_1}$ (25)

We can observe that in both universes $\boldsymbol{x_3}$ is the superior subject which means that in both Combination of attributes and sub-attributes $\boldsymbol{x_3}$ is reflecting the best.

Step 6. Authenticity measure of final Ranking:

Percentage authenticity for first level $\boldsymbol{\alpha}$ universe

By using eq. 7 :

Percentage authenticity of first position for $\boldsymbol{x_3 = 75}\%$

Percentage authenticity of first position for $\boldsymbol{x_2 = 66.67}\%$

Percentage authenticity of first position for $\boldsymbol{x_1 = 100}\%$

Percentage authenticity for first level $\boldsymbol{\beta}$ universe

By using eq. 7 :

Percentage authenticity of first position for $\boldsymbol{x_3 = 100}\%$

Percentage authenticity of first position for $\boldsymbol{x_2 = 100}\%$

Percentage authenticity of first position for $\boldsymbol{x_1 = 100}\%$

Step 7. Final Universal Ranking:

The final ordering of universes provided as.

Maximum Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

taking $\boldsymbol{k = 1, 2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t = 1}$ as described in eq. 8, we can provide maximum universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega_{A^1}^1(X) = 0.7,\ \Omega_{A^2}^1(X) = 0.9} \qquad (26)$$

Where $\boldsymbol{X}$ representing merged subjects, we can see by using operator $\boldsymbol{t = 1}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe.

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Minimum Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

Taking $\boldsymbol{k = 1, 2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t = 2}$ as described in eq.. 9 we can provide minimum universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^{2}_{A^1}(X) = 0.1,\ \Omega^{2}_{A^2}(X) = 0.2} \qquad (27)$$

We can see by using operator $\boldsymbol{t = 2}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe.

Average Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

similarly, taking $\boldsymbol{k = 1, 2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t = 3}$ as described in eq.. 10, we can provide average universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^{3}_{A^1}(X) = 0.425,\ \Omega^{3}_{A^2}(X) = 0.575} \qquad (28)$$

we can see by using operator $\boldsymbol{t = 3}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe

5. Conclusions

1. We can see from expressions 18, 19, 20 the final order in eq. 24 is the most frequently observed order in all these ranking orders which is also observed same in local ordering of β universe (Eq. 21, 22, 23) and is again same in final global ordering of β universe 25, which shows the final global Ranking is transparent and authentic.

2. expressions 26, 27, 28 provides highest, lowest, and average cumulative memberships of universes.

3. In Universal ordering, it is observed that on the Global Universal level, β universe is better than α universe.

1. Local ordering: we can observe local ordering by using the novel plithogenic hyper-super-soft-matrix and local operators. We can judge the performance of some fixed subjects in a particular universe, i.e., one combination or level sheet. This is the case of the inner classification of the universe.

2. Global ordering: We can provide global ordering by considering the performance of some fixed subjects in multiple universes, i.e., different combinations of sub-attribute or level sheets are combined by using the frequency matrix. This is the case of combining several angles of visions in the final decision.

3. Universal ordering: We can compare these universes by combining the cumulative memberships of the last column for each universe. The universe having the largest cumulative average membership is the better and then can write them by descending order.

4. **Extreme Universal Memberships:** We can also find out extreme values of these universes and can judge these subjects in a grand universe which is made by multiple smaller parallel universes so we can choose the best subject from all universes that is the one who is best in most universes or we can select one reality out of multiple parallel realities these facts are useful in the field of artificial intelligence.

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