Objectives

To Inspire, motivate researchers and students working in the areas of applied mathematics and its recent applications.

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Schedule ICRAAM

Day 1: February 20, 2019 (Wednesday)

First Session: Session Chair: Moiz ud Din Khan		
Time	**Talk**	**Speakers**
08:45- 9:30	**Registration and opening ceremony**	
09:30 -10:15	***Stability and Control of Hybrid Impulsive and Switched Dynamical Systems***	**Naim Bajcinca** ***TU Kaiserslautern, Germany***
10:15-10:45	**Tea &Refreshment**	
Morning Parallel Session I Session Chair: Zaighum Zia		
10:45 -11:30	***Theoretical Study of Thermal and Radial Effects in Liquid Chromatography***	**Shamsul Qamar** ***COMSATS University Islamabad***
11:30 -12:00	***Mathematical Model of Boundary Layer Flow in a Porous Medium Filled by a Nanofluid: Stability Analysis***	**Norihan Md Arifin** ***Universiti Putra, Malaysia***
12:00 -12:30	***Gravitational Decoupled Anisotropic Solutions***	**Muhammad Sharif (TI)** **Univ*ersity of the Punjab, Lahore***
12:30 – 13:00	***Trigonometric Hamming similarity operators***	**Samina Mazhar** ***LCWU, Lahore***
Morning Parallel Session II Session Chair: M. Yousaf Bhatti		
10:45 -11:15	***Process of Transportation during Gravitational Collapse***	**Hafiza Rizwana Kausar** ***University of Central Punjab, Lahore***

11:30 -12:00	***N-dimensional Non-vacuum Plane Symmetric Solutions in $f(R,T)$ Gravity***	**Sadia Sattar** ***The University of Lahore, Sub-campus Sargodha***
12:00 -12:30	***Estimation of Different Entropies by using refinement of Jensen's inequality and their Generalization for higher order convex function***	**Tasadduq Niaz** ***The University of Lahore, Sub-Campus Sargodha***
12:30 – 13:00	***Ternary non-stationary subdivision schemes in hyperbolic forms***	**Wardat us Salam** ***University of Education, Lahore***
13:00 -14:15	**Lunch Break**	
Afternoon Parallel Session I Session Chair: Yousaf Habib		
14:15 - 14:50	***Fractional Mechanism of Microwave Radiating Therapy of Breast Cancer***	**Muhammad Imran Asjad** ***UMT, Lahore***
14:50 - 15:15	***H-Irregularity strength for some family of graphs.***	**Muhammad Numan** ***COMSATS University Islamabad, Attock Campus***
15:15 -15:40	***Coupled orbital-thermal evolution of major Uranian satellites***	**Attique Ur-Rehman** ***Virtual University of Pakistan***
15:40 – 16: 05	***A Multicriteria Decision-Making Approach Based on Fuzzy AHP with Intuitionistic 2-Tuple Linguistic Sets***	**Shahzad Faizi** ***Virtual University of Pakistan***
16:05 – 16: 30	***Existence of Wormholes and Noether Symmetry Approach in f(R, T) Gravity***	**Iqra Nawazish** ***University of Education, Lahore.***

Afternoon Parallel Session II Session Chair: Hani Shaker		
14:15 - 14:50	***Plane Symmetric Gravitational Waves Like Spacetimes and the Conservation Laws***	**Farhad Ali** ***Kohat University of Science and Technology,KPK, Pakistan***
14:50 - 15:15	***Existence results for nonlinear coupled boundary value problems***	**Imran Talib** ***Virtual University of Pakistan***
15:15 -15:40	***Compact Stars using TOV Equation in Modified Gauss-Bonnet Gravity***	**Ms. Tayyaba Naz** ***National University of Computer and Emerging Sciences (FAST), Lahore***
15:40 – 16: 05	***Non-flat FRW Universe Version of Tsallis Holographic Dark Energy in Specific Modified Gravity***	**Mr. Nadeem Azhar** ***The Crescent College, Lahore***
16:05 – 16: 30	***MHD influence on different water based nanofluids (T iO2,Al2O3,CuO) in porous medium with chemical reaction and Newtonian heating***	**Ms. Maryam Aleem** ***UMT, Lahore***

Day 2: February 21, 2019 (Thursday)

First Session Session Chair: Norihan Md Arifin		
Time	***Talk***	***Speaker***
08:45 -09:30	***Solving Boundary Value Problem for Delay Differential Equation using Multistep Block Method***	**Zanariah Abdul Majid** ***Universiti Putra, Malaysia***
09:30 -10:15	***Qualitative Criteria for Solutions of Various Kind of Integro -Differential and Impulsive Differential Equations***	**Cemil Tunc** ***Van Yuzuncu Yıl University,Turkey***
10:15 -10:45	**Tea & Refreshment**	
<u>Morning Parallel Session I</u> Session Chair: Kashif Ali		
10:45-11:30	***On fractional derivatives and generalized functions***	**Adem Kilicman** ***Universiti Putra, Malaysia***
11:30 -12:00	***Strong Gravitational Lensing for Photon Coupled to Weyl Tensor in Kiselev Black Hole***	**Ghulam Abbas** ***The Islamia University of Bahawalpur***
12:00 -12:30	***Numerical Analysis for Role of Media and Treatment on an Infectious Disease Dynamics***	**Muhammad Rafiq** ***University of Central Punjab, Lahore***
12:30 – 13:00	***A fractional order HIV-TB co-infection model with non-singular Mittag-Leffler Law***	**Hasib Khan** ***Shaheed BB University, Sheringal, Dir.***

Morning Parallel Session II Session Chair: Muhammad Hussain		
10:45 -11:20	***A Combinatorial Result about Sequential Computation of Function***	**Rana Safdar** ***University of Sargodha, Sargodha***
11:20 -11:45	***Dynamics of Charged Viscous Dissipative Spherical Collapse with Perturbation Approach***	**Syed Munawar Shah** ***The Islamia University of Bahawalpur***
11:45-12:10	***Models of Collapsing and Expanding Anisotropic Gravitating Source in f(R,T) Theory of Gravity***	**Riaz Ahmed** ***The Islamia University of Bahawalpur***
12:10 -12:35	***Gravitational Perfect Fluid Collapse in Gauss-Bonnet Gravity***	**Muhammad Tahir** ***The Islamia University of Bahawalpur***
12:35 – 13:00	***Matter Accretion onto a Brane-World Black Hole Via Hamiltonian Approach***	**Allah Ditta** ***The Islamia University of Bahawalpur***
13:00 -14:15	**Lunch Break**	
Morning Parallel Session III Session Chair: Ayesha Sohail		
10:45 -11:20	***Static Spherically Symmetric Wormholes in Generalized Gravity***	**Farzana Kausar** ***COMSATS University Islamabad, Lahore***
11:20 -11:45	***Accretion onto Charged Black Holes in Various Theories of Gravity***	**Shahid Chudhary** ***Sharif College of Engineering and Technology, Lahore.***
11:45-12:10	***Vertex Anti-magic Total Labeling***	**Muhammad Mubashar** ***COMSATS University Islamabad, Lahore***

12:10 -12:35	***Topological Descriptor of 2-Dimensional SiliconCarbons and Their Applications***	**Nadeem Ghauri** *COMSATS University Islamabad, Lahore*
12:35 – 13:00	***Generators of Maximal Subgroups of Harada-Norton and some Linear Groups***	**Faisal Yasin** *COMSATS University Islamabad, Lahore*
13:00 – 14:15	**Lunch Break**	
Afternoon Parallel Session I Session Chair: Cemil Tunc		
14:15 - 15:00	***Hartmann boundary layer in peristaltic flow for viscoelastic fluid***	**Saleem Asghar,** *COMSATS University Islamabad*
15:00 - 15:30	***Numerical simulations of fractional nonlinear Hartmann flow with revised heat flux model***	**Amer Rasheed** *LUMS, Lahore*
15:30 -16:00	***Functional partial differential equations and cell division***	**Ali AshherZaidi** *LUMS, Lahore*
16:00 – 16: 30	***Continuous approximations for long-term numerical simulations of the solar system***	**Sahfiq Ur Rehman** *UET Lahore. Pakistan*
Afternoon Parallel Session II Session Chair: Imran Asjad		
14:15 - 14:45	***Second-order slip effects on oscillating fractionalized Maxwellfluid in porous medium***	**Muhammad Jamil** *NED University of Engineering & Technology, Karachi*
14:45 - 15:10	***Complexity Factor For Anisotropic Source in Non-minimal Coupling Metric f(R) Gravity***	**Hammad Nazar** *The Islamia University of Bahawalpur*
15:10 -15:35	***A New Model of Quintessence Compact Stars in the Rastall Theory of Gravity***	**Muhammad Rizwan Shahzad** *The Islamia University of Bahawalpur*

15:35 – 16:00	***Aspects of Cosmic Compact Star in f(R) Gravity.***	**Iffat Fayyaz** ***National University of Computer and Emerging Sciences (FAST), Lahore***
16:00 – 16: 30	***About dynamic behavior of rational difference equation***	**Ansar Abbas** ***Riphah International University, Islamabad***
Afternoon Parallel Session III Session Chair: Qurat ul Ain		
14:15 - 14:45	***Numerical comparisons of finite element stabilized methods for high Reynolds numbers vortex dynamics simulations***	**NaveedAhmed** ***LUMS, Lahore***
14:45 - 15:10	***Peristaltic flow of Jeffrey Six Constant Nanofluid Flow in an Endoscope with Non-uniform Vertical Tube***	**Aqila Shaheen** ***CUI, Lahore***
15:10 -15:35	***Multiple Complex Soliton for Nonlinear Telegraph Equation***	**Badar Nawaz** ***COMSATS University Islamabad, Lahore***
15:35 – 16:00	***Topological Characterization of the Symmetrical Structure of Bismuth Tri-Iodide***	**Muhammad Arfan Ali** ***COMSATS University Islamabad, Lahore***
16:00 – 16: 30	***Hosoya Polynomial of Graphs***	**Haseeb Ahmad** ***COMSATS University Islamabad, Lahore***
	M-Polynomial of Chemical Graphs	**Farrukh Ijaz** ***COMSATS University Islamabad, Lahore***

Day 3: February 22, 2019 (Friday)

First Session ***Session Chair: Zanariah Abdul Majid***		
Time	***Talk***	***Speaker***
08:45 -09:30	***Second Derivative General Linear Methods for the Numerical Solution of Ordinary Differential Equations***	**Gholam Reza Hojjati** ***University of Tabriz, Iran***
09:30 -10:15	***A Journey through Structure-Preserving Discretization***	**Raffaele D'ambrosio** ***University of L'Aquila, Italy***
10:15 -10:45	**Tea & Refreshment**	
Morning Parallel Session I Session Chair: Gholam Reza Hojjati		
10:45-11:30	***Numerical Study of Reaction Mechanism of Methane***	**Khalid Saifullah Syed** ***BZU, Multan***
11:30 -12:15	***A method for solving nonlinear fractional partial differential equation***	**Umer Saeed** ***National University of Sciences and Technology (NUST), Islamabad***
12:15 -12:45	***Numerical Computation of Shock Waves by Using Simplified Ghost Point Treatment***	**Muhmmad Asif Farooq** ***National University of Sciences and Technology (NUST), Islamabad***
	Lunch Break	

Morning Parallel Session II Session Chair: Raffaele D'ambrosio		
10:45-11:20	***Fractional Order Derivative Model for Artificial Pancreas***	**Muhammad Umer Saleem** ***University of Education, Lahore***
11:20 -11:45	***Numerical Solution of the SIR Model with Vaccination, Treatment and Incidence Rate Using Galerkin Scheme***	**Atta ullah** ***Bacha Khan University Charsadda, KPK***
11:45 -12:15	***Stability of Charged PSR J1614-2230***	***Syed Ali Mardan Azmi*** ***UMT, Lahore***
12:15 – 12:40	***Solution of Hamilton Jacobi Equations to Constraint a Model with Inflation***	**Rabia Saleem** ***CUI, Lahore, Pakistan.***
12:40 - 14:15	**Jumma Prayer/Lunch Break**	
Afternoon Session Session Chair: Sarfraz Ahmed		
14:15 – 14: 45	***Chaotic Dynamical Systems and the problem of prediction.***	**Stefano Luzzatto** ***Strada Costiera, Trieste, Italy***
14:45 – 15:30	**Closing Ceremony**	

ABSTRACTS

International Invited Speakers

Mathematical Model of Boundary Layer Flow in a Porous Medium Filled by a Nanofluid: Stability Analysis

Norihan Md Arifin
Institute for Mathematical Research, Universiti Putra Malaysia.

Several types of mathematical model in a porous medium filled by a nanofluid will be discussed. In the presence of a thermal radiation, the flow is generated due to stretching or shrinking sheet. The problem is formulated for three types of nanoparticles, namely, copper (Cu), alumina (Al2O3) and titania (TiO2). The boundary layer conservation equations are transformed using appropriate similarity variables and the resulting nonlinear boundary value problem is numerically solved using shooting method. It is revealed that the corresponding results possessed two branches of solutions, where we then implemented an analysis of stability on those two non-unique solutions to evaluate the most realizable solution and the features of the respective solutions have been discussed in details.

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Stability and control of hybrid impulsive and switched dynamical systems

Naim Bajcinca
Technical University of Kaiserslautern, Germany

Hybrid dynamical systems arise whenever continuous and discrete dynamics interact. This is often the case when logic decision making or embedded control actions are combined with continuous physical processes. Indeed, nearly all complex dynamic processes in automation, economy, logistics, production, biology, neuroscience, etc. display such heterogeneity. In the talk we provide an introduction to the analytical approaches and in particular address stability and control of specific classes of hybrid systems including switched systems, impulsive dynamical systems and hybrid Petri nets. Additionally, we discuss applications in cyber physical systems, in particular event-triggered control and briefly discuss the applications in systems biology and neuroscience.

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A Journey through Structure-Preserving Discretization

Raffaele Dambrosio
Department of Engineering, Computer Science and Mathematics, University of L'Aquila, Italy.

In this talk we present recent advances in the numerical approximation of various evolutionary problems by means of methods preserving the qualitative behavior of the operator along the discretized dynamics. This approach is applied to both deterministic and stochastic problems, with a rigorous setting matched with a proper experimental one that confirms the effectiveness of the introduced methodologies. As regards deterministic structure-preservation, we mostly deal with Hamiltonian problems, treated by multi-value numerical methods whose long-term properties are highlighted. Concerning stochastic problems, a possible structure-preserving framework is introduced for stochastic Hamiltonian problems (with the aim of retaining the known long-term properties on the expected Hamiltonian) as well as and stochastic oscillators (in order to reproduce the long-term properties of the position and the velocity of the oscillating particle).

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Second Derivative General Linear Methods for the Numerical Solution of Ordinary Differential Equations

Gholam Reza Hojjati
Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran.

Traditional numerical methods for the numerical solution of an autonomous system of ordinary differential equation:

Y’ = f(y(x)), where y is a function from R to Rm and f is a function from Rm to Rm.

generally fall into two main classes: linear multistep (multi-value) and Runge–Kutta (multistage) methods. In 1966, Butcher introduced general linear methods as a unifying framework for the traditional methods to study the properties of consistency, stability and convergence, and to formulate new methods with clear advantages over these classes. In fact, this class of the methods includes all the first derivative multi-value and multistage methods. On the other hand, to construct methods with higher order and extensive stability region, some efficient second derivative

methods within the class of linear multistep methods and Runge–Kutta methods have been introduced. In 2005, Butcher and Hojjati extended GLMs to the case in which second derivatives, as well as first derivatives, can be calculated. These methods, called SGLMs, were studied more by Abdi and Hojjati. Starting with a discussion on the intrinsic structure of SGLMs, in this talk we will become familiar with the properties of SGLMs and their efficiency in solving of the stiff and non-stiff initial value problems. This is followed by the introducing of some subclasses of SGLMs which preserve qualitative geometrical properties of the flow of the system. Also, some strategies based on SGLMs are given for the numerical solution of conservative problems.

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On Fractional Derivatives and Generalized Functions

Adem Kilicman
Institute for Mathematical Research, Universiti Putra Malaysia.

The fractional calculus is considered as an extension of ordinary derivatives and integrals to arbitrary order possible complex number. The historical development of subject is sufficiently old enough and can go back to the times of Leibnitz and Newton. After the introduction of fractional derivatives idea, almost a three hundred years the fractional calculus was not much popular in science and engineering. However recently become very famous and has been applied broad range of problems in several areas such as engineering, science, finance, as well as bio engineering etc. In fact it was observed that the modelling with fractional order is more natural than the classical calculus. In the literature there are many different types of related definitions due to the properties. In the present study we extend fractional differential calculus to generalized functions also known as distributions by using the infinitely differentiable functions having compact support as test functions. We also define several new distributions by using the fractional derivatives. And provide some examples in applied sciences such as in finance.

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Chaotic Dynamical Systems and the Problem of Prediction.

Stefano Luzzatto
Strada Costiera, Trieste, Italy

Ordinary Differential Equations and Dynamical Systems have been used for hundreds of years to construct models of physical processes. The discovery of the property of “chaos” in real life phenomena and in mathematical models raises significant questions about the extent to which these models can actually help us to understand the evolution of a system. I will discuss, mainly from a mathematical point of view, the notion of chaos and the kinds of systems in which it appears. I will also discuss how recent mathematical developments suggest that a probabilistic and statistical approach might be the best way to handle such chaotic systems.

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Solving Boundary Value Problem for Delay Differential Equation using Multistep Block Method

Zanariah Abdul Majid
Faculty of Science, Universiti Putra, Malaysia

In this study, we propose a multistep block method for the solution of boundary value problem for second order delay differential equations directly. The proposed block method will approximate the solutions at two points simultaneously and will solve the delay differential equations directly without reducing to the system of first order. The shooting technique by using Newton's method will be implemented to compute the guessing values. Some numerical examples are presented to show that the proposed method is capable for solving boundary value problems for delay differential equations.

Qualitative Criteria for Solutions of Various Kind of Integro-Differential and Impulsive Differential Equations

Cemil Tunç
Professor Department of Mathematics, Faculty of Sciences
Van Yuzuncu Yıl University, 65080, Van-Turkey

In this work, a class of non-linear Volterra integro-differential equations, Volterra integro Caputo fractional differential equations with delay and linear periodic impulsive systems with time delay are considered. New criteria are presented on the various qualitative properties of solutions of these equations. The Lyapunov–Krasovskii method, the Razumikhin method and some others methods are used as basic tools. Examples are given to verify the obtained results.

National Invited Speakers

Numerical comparisons of finite element stabilized metho ds for high Reynolds numb ers vortex dynamics simulations

Naveed Ahmed
Department of Mathematics, Syed Babar Ali School of Sciences & Engineering, Lahore University of Management Sciences

In this talk, I will present up-to-date and classical Finite Element (FE) stabilized methods for time -dependent incompressible flows. All studied methods belong to the Variational MultiScale (VMS) framework. So, different realizations of stabilized FE-VMS methods are compared in high Reynolds numbers vortex dynamics simulations. In particular, a fully Residual-Based (RB)-VMS method is compared with the classical Streamline-Upwind Petrov–Galerkin (SUPG) method together with grad-div stabilization, a standard one level Local Projection Stabilization (LPS) method, and a recently proposed LPS method by interpolation. These procedures do not make use of the statistical theory of equilibrium turbulence, and no ad-ho c eddy viscosity modeling is required for all methods. Applications to the simulations of high Reynolds numbers flows with vortical structures on relatively coars e grids are s how cased, by focusing on two-dimensional plane mixing-layer flows. Both Inf-Sup Stable (ISS) and Equal Order (EO) FE pairs are explored, using a second order semi-implicit Backward Differentiation Formula (BDF2) in time. Based on the numerical studies, it is concluded that the SUPG method using both ISS and EO FE pairs performs best among all methods. Furthermore, there seems to be no reason to extend SUPG method by the higher order terms of the RB –VMS method.

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Hartmann Boundary Layer in Peristaltic Flow for Viscoelastic Fluid

Saleem Asghar,
Department of Mathematics, COMSATS University Islamabad, Pakistan.

The phenomenon of peristalsis is studied for the non-Newtonian Jeffrey fluid in an axisymmetric tube under the influence of strong uniform magnetic field. We employ the usual permissible small Reynolds number and large wavelength approximations in the peristaltic transport phenomena. The modeled differential equation is purely singular in nature for large values of Hartmann number. The boundary value problem is solved analytically using singular perturbation approach together with higher order matching technique. The analytical expressions for stream function, velocity, and pressure rise are obtained. The conventional analysis of obtaining the analytic results has been further extended to the considerations of dynamical system of the problem to understand the flow behavior. The time evolution of the fluid flow has been presented in the phase space, bifurcation of the solution has been carried out for magnetic parameter, amplitude ratio and flow rate and the concomitant change of stability is given through topological flow patterns.

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Numerical Computation of Shock Waves by Using Simplified Ghost Point Treatment

Muhmmad Asif Farooq
Dept. Mathematics, National University of Sciences and Technology, Islamabad, Pakistan.

In this work we discuss simplified ghost point treatment to calculate shock waves over immersed object. The scheme was introduced by the author of this paper to easily compute shock waves. We solve compressible flow over immersed bodies while discussing benchmark problems. We find supersonic and subsonic flow over a wedge, circular arc airfoil. We find good agreement from literature.

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Second-Order Slip Effects on Oscillating Fractionalized Maxwell Fluid in Porous Medium

Muhammad Jamil
Department of Mathematics, NED University of Engineering & Technology, Karachi

The objective of this article is to investigate the effects of second order slip on the MHD flow of fractionalized Maxwell fluid through a porous medium due to oscillatory motion of an infinite plate. The governing equations are developed by fractional calculus approach using Caputo-Fabrizio fractional derivative definition. The exact analytical solutions for velocity field and associated shear stress are calculated using Laplace transforms and presented in series form in terms of generalized M-function satisfying all imposed initial and boundary conditions. The flow of fractionalized Maxwell fluid with and without slips, in the presence and absence of magnetic effect, the solutions for ordinary Maxwell and Newtonian fluid performing the similar motion are derived as the limiting cases. The impact of fractional parameter, magnetic, slip coefficients, and porosity parameter over the velocity field and shear stress are discussed and analyzed through graphical illustrations.

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Process of Transportation during Gravitational Collapse

Hafiza Rizwana Kausar
University of Central Punjab, Lahore, Pakistan.

In this talk, we present the transport equation which provides the information about the transfer of mass, heat and momentum during the gravitational collapse of massive stars. We adopt the modified theories of gravity and examine the process of energy transport and its effects on the collapsing process. We discuss how such theories may incorporated mathematically into the analysis and control of the dynamics of a complex system.

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Trigonometric Hamming Similarity Operators

Samina Mazhar
Dept. Mathematics, LCWU, Lahore, Pakistan.

This talk is aimed to a present several new similarity measures based on trigonometric Hamming similarity operators of rough neutrosophic sets and their applications in decision making. Some properties of the proposed similarity measures are established. Also, a numerical example will be given to illustrate the applicability of the proposed similarity measures in decision making.

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A method for Solving Nonlinear Fractional Partial Differential Equations

Umer Saeed
Department of Mathematics, National University of Sciences and Technology Islamabad

The purpose of the presentation is to propose a method for solving nonlinear fractional partial differential equations on the semi-infinite domain, and to get better and more accurate results The operational matrices for the method will be derived, constructed and utilized for the solution of nonlinear fractional partial differential equations. The operational matrices contain many zero entries, which lead to the high efficiency of the method and reasonable accuracy is achieved even with less number of grid points. Many Engineers can utilize the presented method for solving their nonlinear fractional models.

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Gravitational Decoupled Anisotropic Solutions

Muhammad Sharif (TI)
Department of Mathematics, University of the Punjab, Lahore Pakistan

The purpose of this paper is to obtain exact solutions for charged anisotropic spherically symmetric matter configuration. For this purpose, we consider a known solution for the isotropic spherical system in the presence of the electromagnetic field and extend it to two types of anisotropic charged solutions through gravitational decoupling approach. We examine the physical characteristics of the resulting models. It is found that only first solution is physically acceptable as it meets all the energy bounds as well as the stability criterion. We conclude that the stability of the first model is enhanced with the increase of charge.

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Numerical Study of Reaction Mechanism of Methane

Khalid Saifullah Syed
Mathematics Department, BZU, Multan, Pakistan

The main purpose of the work we done is to study the reaction mechanism of methane and its properties and get an understanding of combustion processes to come to the point where we can reduced the full mechanism by constraining it to our desired situation. We take one dimensional premixed laminar (when a fluid flows in parallel layers, with no disruption between the layers) flames. Many practical combustors, such as internal combustion engines, rely on premixed flame propagation. Laminar flame speed is often used to characterize the combustion of various fuel-oxidizer combinations and in determining mixture flammability limits. Therefore, the ability to model chemical kinetics and transport processes in these flames is critical to flammability studies, interpreting flame experiments and to understanding the combustion process itself. Examples of the use of flame modeling to interpret experimental observations and to verify combustion chemistry and pollution formation can be found in Miller, et al.

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Theoretical Study of Thermal and Radial Effects in Liquid Chromatography

Shamsul Qamar
Department of Mathematics, COMSATS University Islamabad,
Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany.

Chromatographic models contain convection-dominated systems of partial differential equations coupled with some differential and algebraic equations describing thermodynamic and kinetic phenomena. Thermal effects are discussed widely in the case of gas chromatography. In liquid chromatography, such effects are typically neglected. The main goal of this work is to quantify how temperature gradients can influence conversion and separation in liquid chromatography. The coupling of concentration and thermal fronts are illustrated and key parameters influencing the column performance are identified. In contrast to previous studies, this work also includes the possibility that radial concentration profiles can develop. For that purpose, two-dimensional models of liquid chromatography are derived in cylindrical geometry. The considered radial gradients are typically ignored, which can be problematic, e.g. in the cases of non-perfect injections and larger column dimensions. To derive analytical solutions the assumption of linear adsorption isotherm is used. The Laplace and Hankel transformations are simultaneously applied to derive analytical solutions of linear models. To further analyze the effects of different kinetic parameters on the elution profiles, statistical temporal moments are derived from the Hankel-Laplace domains solutions. In the case of nonlinear isotherms, a high resolution flux limiting finite volume scheme is applied to solve the model equations. Several case studies are carried out to analyze the effects of different parameters on the elution profiles. The developed analytical and numerical solutions could be useful for improving the process and for the estimation of model parameters from results of laboratory-scale experiments.

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Numerical Simulations of Fractional Nonlinear Hartmann Flow with Revised Heat Flux Model

Amer Rasheed
Mathematics Department, LUMS Lahore.

In this communication we have examined the magneto-hydrodynamic flow of viscoelastic liquid with revise version of thermal flux. Thermal conductivity of the fluid is variable. Effects of body forces such as gravity are also examined in terms of convection. Non-integer time derivatives are used for the better understanding of viscoelastic flow characteristics. In literature, these derivatives have been proved suitable for the predictions of hereditary systems such as viscoelastic flows. Heat transfer is analyzed in the flow field via modified fractional thermal gradient. Temperature gradient exists between the flow boundaries and increases with the passage of time. Flow is generated by the variable movement of flow boundary. All the segments of numerical modeling are completely addressed in this paper i.e. physical system, mathematical model, simulation, prediction, validation and verification. Unsteady motion of in-compressible fluid is directed by differential equations with fractional partial derivatives, for the solutions of these equations numerical techniques are necessary to use. Governing equations with appropriate flow conditions have been discretized by finite difference along with finite element algorithms. Theoretical error appraisals are hypothesized and created numerical plan has been approved by doing error investigations numerically regarding spatial directions. We have shown the estimate error and plotted the error curves in logarithmic scales of $L_2(\Omega)$ and $H^1(\Omega)$ norms by taking absolute approximations of the space discretization. Local Nusselt number and coefficients of skin friction are determined for non-integer viscoelastic model. Flow field is simulated for various values of physical parameters. The obtained results showed that fractional exponent α has opposite effects on the velocity, temperature profiles for time $t = 2$. It is also noted that temperature gradient increases by the increase of fractional number α and thermal relaxation parameter γ. Viscoelastic flows in polymer and plastics industries can be addressed by similar technique.

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Functional Partial Differential Equations and Cell Division

Ali Ashher Zaidi
Mathematics department, LUMS Lahore.

A simple model for cell growth and division into $\alpha > 1$ daughter cells is given by the functional PDE

$$-\frac{\partial^2}{\partial x^2}\left(D(x)n(x,t)\right)+\frac{\partial}{\partial x}\left(g(x)n(x,t)\right)+\frac{\partial}{\partial t}n(x,t)+bn(x,t)=b\alpha^2 n(\alpha x,t)$$

Here, n denotes the number density of cells of size x at time t, D is dispersion, g is the growth rate, and b is the division rate. ("Size" is usually measured by mass or DNA content) The differential equation is supplemented by the condition

$$n(x,0)=n_o(x)$$

where n_o is the initial cell size distribution, and the boundary condition

$$-\frac{\partial}{\partial x}\left(D(x)n(x,t)\right)+g(x)n(x,t)=0$$

The problem is thus of the initial-boundary value type. There is a paucity of analytical solution techniques for these problems; however, it is possible to solve the problem for some simple cases of interest. Although the leading order long time asymptotic behavior of solutions to these problems is known for fairly general cases, the higher order terms are relatively unexplored. The exact solutions yield the higher order long time asymptotic behavior of solutions for the special cases and may provide some insight into more general cases.

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Contributory Talks

About Dynamic Behavior of Rational Difference Equation

Ansar Abbas
Riphah International University, Islamabad, Pakistan.

To study the global stability character and the periodic nature of the solutions of the Difference equation

$$x_{n+1} = ax_{n-l} + \frac{b + cx_{n-k}}{dx_{n-s} + ex_{n-t}}, n = 0,1,2...$$

Where the initial condition $x_{-r}, x_{-r-1}, ..., x_o$ are arbitrary positive real numbers, $r = \{l, k, s, t\}$ is non-negative integer and a, b, c, d, e positive constants. Finally, some numerical examples are presented and graphed by MATLAB.

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Weak Gravitational Lensing and Gauss-Bonnet Theorem

Jameela Abbas
University of Education, Lahore

The phenomenon of weak gravitational lensing is studied. In our analysis, we find the weak gravitational lensing by using Gauss-Bonnet theorem to a suitable osculating Riemannian manifold. Latter, for verification of obtained results, the weak gravitational lensing is illustrated along the geodesic formalism. The graphical analysis for deflection angle also analyzed.

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Topological Characterization of the Symmetrical Structure of Bismuth Tri-Iodide

Muhammad Arfan Ali
COMSATS University Islamabad, Lahore, Pakistan.

The bismuth tri-iodide is an inorganic compound. It is the result of the response of bismuth and iodine, which created enthusiasm for subjective inorganic investigation. The topological indices are the numerical invariants of the molecular graph that portray its topology and are normally graph invariants. In 1975, Randic presented in a bond-added substance a topological index as a descriptor for portraying sub-atomic branching. Here, we investigate the precious stone structure of bismuth tri-iodide chain and sheet. Moreover, the exact formulas of degree based added substance topological lists principally the first, second and hyper Zagreb index, the general Randic index, the geometric-arithmetic index, the fourth atom bond connectivity and the fifth geometric arithmetic index of sub-atomic graph of bismuth tri-iodide for both chain and sheet structures are~determined.

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Strong Gravitational Lensing for Photon Coupled to Weyl Tensor in Kiselev Black Hole

Ghulam Abbas
Department of Mathematics, The Islamia University of Bahawalpur

The ambition of this paper is to highlight the experience of the strong gravitational lensing and deflection angle for the photons coupling with Weyl tensor in a Kiselev black hole spacetime. However, we first present photon equations of motion coupled to Weyl tensor, which brings light cone conditions. We investigate that the gravitational lensing depends not only on the parameter α, but it also depends on the coupled photon polarization directions. We formulate the critical values α_{c1} for PPM and α_{c2} for PPL, for the existence of the photon sphere radius r_{ps} outside the event horizon, which depends on the photon polarization directions and the quintessence

parameter σ. Furthermore, we find that the polarization directions of coupled photon and the coupling parameter α both modify the features of the photon sphere, the angle of deflection and the functions $\bar{a}$ and $\bar{b}$ in the strong gravitational lensing in a Kiselev black hole spacetime. Also, we have discussed the shadow of this particular black hole and magnification factor in the strong gravitational lensing in detail.

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Hosoya Polynomial of Graphs

Haseeb Ahmad
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In the fields of chemical graph theory, topological index is a type of a molecular descriptor that is calculated based on the graph of a chemical compound. In 1947 Harry Wiener introduced "Path number" which is now known as Wiener index and is the oldest topological index related to molecular branching. Hosoya polynomial plays a vital role in determining Wiener index. The Hosoya polynomial of G is $H(G,X)=\frac{1}{2}\sum_{u,v\in V(G)} x^{d(u,v)}$where $d(u,v)$denotes the distance between vertices u and v. The Wiener index is the sum of distances between all pairs of vertices of a connected graph G as $W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d(u,v)$.

==========

Plane Symmetric Gravitational Waves Like Spacetimes and the Conservation Laws

Farhad Ali
Department of Mathematics, Kohat University of Science and Technology, Kohat, Pakistan

Some classes of plane symmetric gravitational waves like spacetimes are presented over here. These are the spacetimes for which the exact form and the time conformal form of the Lagrangians admit the same numbers of Noether symmetries. The Noether symmetries of the Lagrangians of the time conformal spacetimes have some nontrivial approximate parts. For each Noether symmetry there exist a conservation law, therefore for the approximate Noether symmetries there exist approximate conservation laws. The approximate part of the conservation law is the correction to the conservation laws in the given spacetime.

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Fractional Mechanism of Microwave Radiating Therapy of Breast Cancer

Muhammad Imran Asjad
Mathematics department, University of Management and Technology, Lahore

This study deals with breast cancer therapy with fractional derivative in porous medium. A moderate temperature hyperthermia treatment is apply which results into cell death due to an increase in the level of cell sensitivity to radiation therapy and blood flow in tumor and oxygen. The classical model of energy balance equation can be generalized to some fractional derivatives. Fractional derivative of Caputo (C) with singular kernel and Atangan-Baleanu (ABC) with non singular kernel are used to develop the fractional model of energy balance equation. We have found the semi exact solutions of initial value problem via Laplace transform. A moderate temperature hyperthermia treatment is apply which results into cell death due to an increase in the level of cell sensitivity and blood flow in tumor and oxygen. Graphical illustration is provided for numerical values of embedded parameters with the help of MATHCAD software. As a result, a very short time is required the therapeutic temperature point to get the tumor cell. A comparison has been drawn between the solutions modeled with fractional derivatives and found that Atangana-Baleanu fractional model is well suited in exhibiting the memory effect of the temperature function.

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Molecular Descriptor of Line Graph of Carbon Nano Cones.

Muhammad Asif
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Let molecular graph as G=(V,E) with V (G) and E(G) as vertex set and edge set respectively. Vertices and edges of G are corresponding to the atom and chemical bonds of molecule, respectively. We denote connection number of vertex u (number of vertices at distance two from u) as σ_u. Zagreb indices are the most studied topological indices as a tool in QSAR\QSPR to predict the molecular properties of the chemical compounds which are correlated to their chemical structure. Recently, modified versions of Zagreb indices named as connection Zagreb indices studied in [1]. In this work, we compute the three Zagreb connection indices ZC_1 (G)=∑_(u∈V(G)) (σ_u)^2 ,ZC_2 (G)=∑_(uv∈E(G)) ⟦σ_u σ_v ⟧ and ZC_1^*=∑_(u∈V(G)) (d_u σ_u)^2 of polycyclic aromatic hydrocarbons.

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MHD Influence on Different Water based Nanofluids (TiO2,Al2O3,CuO) in Porous Medium with Chemical Reaction and Newtonian Heating

Maryam Aleem
UMT, Lahore, Pakistan.

The present study is aimed to provide the unsteady MHD nanofluid's flow passing through an accelerating infinite vertical plate situated in porous medium. The flow is effected by thermal radiation, Newtonian heating and chemical reaction. Water is considered as conventional base fluid comprising of five different types of nano particles such as Titanium oxide (T iO2), Aluminium Oxide (Al2O3), Copper Oxide (CuO), Silver (Ag) and Copper (Cu). By using dimensional analysis, the governing equations for temperature, velocity and concentration are reduced to dimensionless and after that these classical equations of present model are generalized to Caputo fractional derivative. Semiexact solutions for these equations are obtained via Laplace transform method. Inversion algorithms (Tzou and Stehfest) are applied to find the inverse Laplace transform. At last the comparison of water based nanofluids suspended with five different types of nano particles is drawn and effect of nanoparticles as well as fractional parameters (α, β, γ) on temperature and velocity can be seen by software Mathcad. We concluded that Ag-water nanofluid has greater temperature due to its greater value of thermal conductivity as compare to others. Whereas Al2O3-water has greater velocity because these particles are less denser than T iO2, Cu, Ag, CuO. Further we can see that by increasing the value of fractional parameters velocity as well as temperature decreases.

==========

Stability of Charged PSR J1614-2230

Syed Ali Mardan Azmi
Dept. Mathematics, University of Management and Technology, Lahore, Pakistan.

We present the stability of compact object PSR J1614-2230 in the quadratic regime with electromagnetic field. We apply the local density perturbation to the hydrostatic equilibrium equation as well as all the physical variables involve in the model. We plotted the force distribution function against the different values of radius and model's parameters both with and without charge. It is observed that PSR J1614-2230 remains stable in neutral case, while PSR J1614-2230 exhibit cracking when charge is introduced. Further, we note that stability region decreases as amount of charge increases.

==========

Models of Collapsing and Expanding Anisotropic Gravitating Source in f(R,T) Theory of Gravity

Riaz Ahmed
Department of Mathematics, The Islamia University of Bahawalpur Pakistan

In this paper, we have formulated the exact solutions of the non-static anisotropic gravitating source in f(R,T) gravity which may lead to expansion and collapse. By assuming the no thermal conduction in gravitating source, we have determine parametric solutions in f(R,T) gravity with non-static spherical geometry filled with anisotropic fluid. We have examined the range of parameter for which expansion scalar become negative and positive leading to collapse and expansion, respectively. Further, using the definition of mass function the condition for the trapped surface have been explored and it has been investigated there exists a single horizon in this case. The impact of coupling parameter λ has been discussed in detail in both cases. For the various values of coupling parameter λ, we have plotted energy density, anisotropic pressure and anisotropic parameter in case of collapse and expansion. The physical significance of the graphs has been explained in detail.

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Non-flat FRW Universe Version of Tsallis Holographic Dark Energy in Specific Modified Gravity

Nadeem Azhar
Department of Mathematics, The Crescent College, Shadman, Lahore. Pakistan.

Among various dark energy models, Tsallis holographic dark energy model shows the dynamical enthusiasm to describe the transition phase of universe. In present paper, we consider Tsallis holographic dark energy with event and apparent horizon as an infrared cutoff in the framework of dynamical Chern-Simons modified gravity and non-flat FRW universe. We explore Hubble, equation of state and deceleration parameters and found that Hubble parameter lies in the range 80^{+60}_{-60} and 120^{+40}_{-40} for event and apparent horizon trajectories, respectively. It is mentioned here that the equation state parameter lies within the range -1.3 ∓ 0.4 (event) and -1.125 ∓ 0.125 (apparent). Also, deceleration parameter for both cases show accelerated and decelerated phase of universe as well as cosmological constant. Moreover, we also checked the stability of our model through square speed of sound, which shows the positive behavior (exhibits the stability of the model). Finally, we observe that the generalize second law of thermodynamics.

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Nonlinear Partial Differential Equations and their Exact Solutions

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Nonlinear evolution equations have significance in nonlinear sciences, especially in applied mathematics and physics. The study of nonlinear evolution equations is an absolute necessity in theoretical physics, since these equations form the essential element in this area of research. The investigation of the exact solutions, like solitary wave, periodic wave and rational function solutions of nonlinear evolution equations play a vital role in description of nonlinear physical phenomena. Last two decades have witnessed a considerable gain in number of new methods for solving nonlinear partial differential equations. These methods offer solutions with free parameters that may be important for explaining some physical phenomena in mathematical physics. Exact solutions allow the researchers to design and run experiments by creating appropriate natural conditions to determine these parameters. Construction of exact solutions might pave the way for discussion among the community of researchers for further developments.

==========

Accretion onto Charged Black Holes in Various Theories of Gravity

Shahid Chudhary
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The accretion process is being investigated onto some important black holes such as Born-Infeld-anti-de sitter black hole, non-linear charged black hole solution in AdS space-time and Einstein-Yang-Mills massive gravity in the presence of Born-Infeld nonlinear electrodynamics. We find out the relations of radial velocity, energy density and change of mass for mention black holes and analyze their behavior graphically for different values of equation of state parameters ω. We also examine the relations for critical speed for these black holes. It is observed that for different state parameters different fluids exhibit different evolutions in black holes backgrounds. The energy density of some fluids is negative or positive near the black hole while other fluids become cause to increase or decrease in black hole mass.

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Mr. Allah Ditta
Department of Mathematics, The Islamia University of Bahawalpur, Pakistan

The falling of a matter onto a black hole (BH) is one of the most interesting astrophysical phenomenon. The purpose of our present paper is to study the matter accretion onto dark matter inspired Brane-world BH. By adopting the Hamiltonian approach, we investigate the accretion process onto Brane-Wolrd BH in the presence of cosmological parameter α and the dark matter parameter β. We formulated the general solutions of accretion using the isothermal equation of state according to the models of BH, further the steady state and spherically symmetric matter flow has been considered here. Particularly, in this paper we have analyzed the accretion process when different test fluids are falling onto a BH. The most interesting aspect of this paper is to discuss the effects parameters of BH α and β on critical matter flow and the mass accretion rate, the detail of the results have been presented graphically. Finally, it has been shown that the parameters α and β play a dominant role for the maximum accretion rate.

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Aspects of Cosmic Compact Star in f(R) Gravity.

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National University of Computer and Emerging Sciences (FAST), Lahore, Pakistan

The f(R) theory is one of the significant modified theories of gravity which has been developed by replacing Ricci scalar R with a function of a Ricci scalar R. The current work deals with the physical attributes and dynamics of anisotropic compact stars in the background of this theory. The set of solutions satisfies all the physical requirements of the realistic stars and eventually indicates the possibility to describe compact objects. The physical behavior of these stars has been analyzed with the observational data. In this setting, we have checked all the regularity conditions and stability of the compact star.

==========

A Multicriteria Decision-Making Approach Based on Fuzzy AHP with Intuitionistic 2-Tuple Linguistic Sets

Shahzad Faizi
Virtual University of Pakistan

In the modern literature related to linguistic decision-making, the 2-tuple linguistic representation model and its useful applications in various fields have been extensively studied and used during the last decade. Recently, some useful multicriteria decision-making (MCDM) methods have been introduced based on fuzzy analytic hierarchy process (AHP) for 2-tuple linguistic representation model. By keeping in mind the importance of this linguistic model, in this paper, we introduce a fuzzy AHP methodology for intuitionistic 2-tuple linguistic sets (I2TLSs) which

is a useful extension of the 2-tuple linguistic representation model. This study is comprised of four stages. In the first stage, we define some operational laws for I2TL elements (I2TLEs) and prove some related important properties. In the second stage, intuitionistic 2-tuple linguistic preference relation (I2TLPR) and multiplicative I2TLPR are defined using I2TLSs. In the 3rd stage, a transformation mechanism is introduced which can transform an I2TLPR to a corresponding intuitionistic preference relation (IPR) and vice versa. In the fourth stage, an approach is proposed for checking the consistency of an I2TLPR and presented a method to repair the inconsistent one by using the proposed transformation mechanism. Finally, a numerical example is given and comparative analysis is carried out with the TOPSIS method to verify the validity of the proposed method.

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Topological Descriptor of 2-Dimensional Silicon Carbons and Their Applications

Nadeem Ghauri
CUI, Lahore, Pakistan.

The latest developments in algebra and graph theory allow us to ask a natural question, what is the application in real world of this graph associated with some mathematical system? Groups can be used to construct new non-associative algebraic structures, loops. Graph theory plays an important role in various fields through edge labeling. In this paper, we shall discuss some applications of bipartite graphs, related with Latin squares of Wilson loops, such as metabolic pathways, chemical reaction networks, Routing and Wavelength Assignment problem, missile guidance, astronomy and x-ray crystallography.

==========

M-Polynomial of Chemical Graphs

Farrukh Ijaz
Department of Mathematics, COMSATS University Islamabad, Lahore

Topological index is a numeric parameter which describe some silent features of a molecular graph representing some chemical compounds. These indices characterize topology and usually hold mathematical properties of graph invariant. These descriptors are intensively used in QSAR analysis whichgive insight to biological properties of chemical compound. We will investigate Zagreb indices through M − polynomial, all versions of ABC indices, Geometric-Arithmetic index (GA), Randic index Rα(G), where $\alpha \in R$ and Harmonic index for subdivision of chemical networks to estimate their chemical properties.

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Analysis of Anisotropic Universe through Dark Energy Model

Nimra Irshad
University of Education, Lahore, Pakistan

We consider a generalized anisotropic universe model Einstein's theory of relativity to examine the current accelerated expansion of universe. In the presence of dark energy model, the behavior of cosmological parameters is investigated. The graphical behavior of these parameters provides consistency with current observations. The trajectories of speed of sound ensure the stability of reconstructed anisotropic models.

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A Fractional Order HIV-TB Co-infection Model with Non-singular Mittag-Leffler Law

Hasib Khan
Department of Mathematics, Shaheed BB University, Sheringal, Dir , Pakistan.

The biological models for the study of Human Immunodeficiency Virus (HIV) and its advanced stage Ac-quired Immune Deficiency Syndrome (AIDS), have been widely studied in last two decades. HIV virus can be transmitted

by different means including; blood, semen, pre-seminal fluid, rectal fluid, breast milk and many more. Therefore, protection and timely treatment are helpful to be recovered. It has been analyzed that patients with HIV have more weaker immune system and are susceptible to infections. Keeping the importance of the HIV models, we are interested to consider an analysis of HIV-TB co-infected model in the AB-fractional differential form. The model is studied for the existence, uniqueness of solution, Hyers-Ulam (HU) stability and numerical simulations with assumption of specific parameters.

==========

Static Spherically Symmetric Wormholes in Generalized Gravity

Farzana Kausar
CUI, Lahore

We have examined static spherically symmetric wormhole solutions in generalized gravity. To do this, we consider three different kind of fluids: anisotropic, barotropic and isotropic. We explore different models and inspect the energy conditions for all of those three fluids. It is found that under some models in this theory, it is possible to obtain wormhole solutions without requiring exotic matter. From our results and for our cases, we conclude that for anisotropic and isotropic fluids, realistic wormhole geometries satisfying the energy conditions can be constructed.

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Multiple Complex Soliton for Nonlinear Telegraph Equation

Badar Nawaz
CUI, Lahore

This presentation studies the soliton solutions for nonlinear Telegraph equation (NLTE) also known as damped wave equation. We will study the one, two, three and N-soliton solutions for this equation with the help of Hirota bilinear method.

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Complexity Factor for Anisotropic Source in Non-minimal Coupling Metric f(R) Gravity

Hammad Nazar
Department of Mathematics, The Islamia University of Bahawalpur Pakistan

In this outline we recognize the idea of complexity factor for static anisotropic self-gravitating source with generalized f(R) metric gravity theory. In present consideration, we express the Einstein field equations, hydrostatic equilibrium equation, the mass function and physical behavior of f(R) model by using some observational data of well-known compact stars like 4U1820-30, SAXJ1808.4-3658 and HerX-1. We define the scalar functions through the orthogonal splitting of the Reimann-Christofell tensor and then find the vanishing complexity condition for self-gravitating system with the help of these scalars. It has been found that the vanishing condition for the complexity are pressure anisotropy and energy density inhomogeneity must cancel each other. Moreover, we study the momentous results of an astral object for the vanishing of complexity factor. Finally, these solutions reduced to previous investigation about complexity factor in General Relativity by taking $\boldsymbol{\lambda}$ =0.

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Existence of Wormholes and Noether Symmetry Approach in f(R, T) Gravity

Iqra Nawazish
Department of Mathematics, University of Education, Township Campus, Lahore. Pakistan.

To investigate wormhole solutions of spherically symmetric space-time via Noether symmetry approach in *f(R, T)* gravity, the *f(R, T)* model appreciating in-direct curvature-matter coupling is chosen and examine symmetry generators, corresponding conserved quantities. The possibility for existence of realistic wormhole solutions for both

dusts as well as non-dust distributions is discussed. The wormhole solutions are constructed through constant and variable forms of red-shift and shape functions. Through graphical analysis of these solutions, the physical existence of wormhole solutions as well as stability/instability are analyzed which helps to study the behavior of energy conditions like null/weak relative to ordinary matter and effective energy-momentum tensors.

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Compact Stars using TOV Equation in Modified Gauss-Bonnet Gravity

Tayyaba Naz
National University of Computer and Emerging Sciences, Lahore

The aim of this paper is to study the compact stars in the context of f(G) gravity. Here we discuss the Tolman Oppenheimer Volkoff equation in modified Gauss-Bonnet gravity and later we will discuss the physical attributes of the compact star like energy density, pressure and mass of the stars through polytropic equation of state by considering the model f(G)= , where being the arbitrary constant.

==========

High Irregularity Strength for Some Family of Graphs.

Muhammad Numan
Department of Mathematics, COMSATS University Islamabad, Attock Campus. Pakistan.

Total edge irregularity strength of G has been already widely studied in many papers. The total a-labeling is said to be a total edge irregular a-labeling of the graph G if for every two different edges e_1 and e_2, it holds $w(e_1)$ not equal to $w(e_2)$, where $w(uv) = f(u) + f(uv) + f(v)$, for $e = uv$. The minimum a for which the graph G has total edge irregular a-labeling is called the total edge irregularity strength of G, denoted by tes(G). A natural extension of this concept is by considering the evaluation of the weight is not only for each edge but we consider the weight on each sub-graph H of G. We extend the notion of the total a-labeling into a total H-irregular a-labeling. The total a-labeling is said to be a total H-irregular a-labeling of the graph G if for H sub-graph of G, the total H-weights Wt(H) are distinct. The minimum a for which the graph G has a total H-irregular a-labeling is called the total H-irregularity strength of G, denoted by tHs(G). In this presentation we study about the total H-irregularity strength for some families of graphs.

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Estimation of Different Entropies by using Refinement of Jensen's Inequality and their Generalization for Higher Order Convex Function

Tasadduq Niaz
Dept. Mathematics, The University of Lahore, Sub-Campus Sargodha, Pakistan.

Jensen's inequality is important to obtain inequalities for divergence between probability distribution. In this talk it is given that by applying a refinement of Jensen inequality and introducing a new functional based on f-divergence functional we obtain some estimate for the new functional, the f-divergence and Renyi divergence. Some inequalities for Renyi and Shannon estimates are constructed. Zipf-Mandelbrot law is used to illustrate the result. Zipf-Mandelbrot law and hybrid Zipf-Mandelbrot law are used to estimate the Shannon Entropy. And generalize the refinement of Jensen's inequality and new inequalities of Renyi Shannon entropies for m-convex function using Montgomery identity.

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Vertex Anti-magic Total Labeling

Muhammad Mubashar
COMSATS University Lahore Campus, Pakistan.

Let G = (V, E) be a graph with v vertices and e edges. An (a; d)-vertex-antimagic total labeling is a bijection f from (V (G))U(E(G)) the set of consecutive integers 1,2,…,v + e such that the weights of the vertices from an arithmetic progression with the initial term a and common difference d. If f (V (G)) ={1,2,…,v} then we call the labeling a (a; d)-vertex-antimagic total labling on Harary graphs. In this paper we formulated vertex anti-magic total labeling of Harary graph as well as vertex anti-magic total labeling on isomorphic copies of Harary graphs.

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Numerical Analysis for Role of Media and Treatment on an Infectious Disease Dynamics

Muhammad Rafiq
University of Central Punjab Lahore, Pakistan.

Numerical Analysis for Role of Media and Treatment on an Infectious Disease Dynamics Numerical analysis involves construction, implementation and analysis of reliable numerical schemes to solve continuous models. These schemes are constructed with the aim that the numerical model must preserve all the essential features of the continuous dynamical system. In this paper, the impact of awareness programs and treatment on the dynamics of an infectious disease has been analyzed numerically. A novel unconditionally stable Non-Standard Finite Difference (NSFD) numerical scheme is proposed and its convergence analysis is presented. Numerical experiments are performed and results are compared with standard finite difference schemes being already used to handle such problems. These schemes are conditionally convergent and may diverge for certain values of discretization parameter. The proposed numerical scheme is dynamically consistent with the biological nature of the continuous model and preserves all of its essential properties.

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Continuous Approximations for Long-Term Numerical Simulations of the Solar System

Sahfiq Ur Rehman
UET,Lahore. Pakistan

We present and analyse the performance of different combinations of four higher-order numerical integrators and up to nine interpolation schemes applied to the problem involving the Sun and four Gas-giants (outer planets), namely, Jupiter, Saturn, Uranus, and Neptune.

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Coupled Orbital-Thermal Evolution of Major Uranian Satellites

Attique Ur-Rehman
Virtual University of Pakistan

We use N-body gravitational simulations to study the long-term evolution of the major Uranian satellites. In this study, we develop a coupled orbital-thermal model of Uranus and its major satellites. The orbital part models the evolution of the satellites' orbital elements with the help of detailed equations of motion and associated perturbing terms. The thermal part models the evolution of the satellites' interior using the heat equation. The model exchanges the feedback between the two parts during the entire simulation. The implementation of our model is based on the numerical integration of developed equations of motion and the heat equation.In this presentation, we will talk about the development and implementation of our model. We will present and discuss the results of the numerical testing of our model.

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Vertex Anti-magic Total Labeling

Mubashar Riaz
Department of Mathematics, COMSATS University Islamabad, Lahore

In any connected graph S the shortest path has length d(s, h) between any two vertices s, h ϵ V(S) then d(s, h) is named as distance between s and h. A vertex s is said to resolve p, w if d(s, p) ≠ d(s, w). Let U = $\{s_1, s_2, s_3, \dots, s_l\}$ be any ordered subset of V(S), h ϵ V(S) as representation possessed by m corresponding to U is symbolized by $r(m|U)$ is the l- tuple $(d(m|s_1), d(m|s_2), d(m|s_3), \dots, d(m|s_l))$. If distinct vertices possess distinct representation for U then U is a resolving set. S possesses metric dimension, symbolized by dim(S), is the lowest cardinality among resolving sets of S. In this research work we investigated metric dimension (M.D) of some graph families e.g. s-mid(G), s-T(G) and middle tower graph. All these graphs have constant metric dimension.

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A Combinatorial Result about Sequential Computation of Function

Rana Safdar
University of Sargodha, Sargodha, Pakistan

Improving speed and way of computation contribute to deal with the demand of modern technology. A novel technique of computation has been introduced which uses no extra variables other than the variables available at the initial stage. Number of steps and way of computing a two dimensional function is introduced in {Ahmad, M. (2011). Memory optimization strategies for linear mappings and indexation-based shared documents, (Doctoral dissertation, Université Henri Poincaré-Nancy I)}. In this paper, we extend this idea to an n dimensional linear mapping. Moreover, we apply this technique for n-dimensional inverse linear mapping. By constructing examples, we verify and implement results successfully.

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Solution of Hamilton Jacobi Equations to Constraint a Model with Inflation

Rabia Saleem
Department of Mathematics, COMSATS University Islamabad, Lahore Campus

An inflationary model is being developed during logamediate and exponential eras in the context of non-minimal coupling. A general formalism is provided using a generalized form of coupling parameter. The r-n_s trajectories are plotted by constraining the model parameters.

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N-dimensional Non-vacuum Plane Symmetric Solutions in $f(R,T)$ Gravity

Dr. Sadia Sattar
Department Mathematics, The University of Lahore, Sub-campus Sargodha, Pakistan.

This work contains the study of the N-dimensional non-vacuum static plane-symmetric spacetimes solutions in $f(R,T)$ theory of gravity by using metric approach. For this purpose more general class of $f(R,T)$ model ,i.e., $f(R,T) = f_1(R) + f_2(T)$ is used. Here, also we make the assumption that $F(R) \propto fR^q$, where f and q are arbitrary constants. To find the solutions we assume the dust case with $p = 0$. The field equations are solved by assuming exponential and power law forms of metric coefficient. Moreover, we have evaluated the energy densities and corresponding functions of $f(R,T)$ model.

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Effects of Thermal Fluctuations on Non-minimal Regular Magnetic Black Hole Fractional Order Derivative Model for Artificial Pancreas

Muhammad Umer Saleem
Department of Mathematics, University of Education Lahore, Pakistan.

In this presentation, Fractional-order derivatives are discussed for a comprehensive glucose insulin regulatory model. Observer is designed for approximating the structure of a blood glucose-insulin with glucose rate disorder to show the complete dynamics of the glucose-insulin system where the fractional-order is $\alpha \in (0, 1]$. The developed method provides the estimation algorithm for a glucose-insulin system with unknown time-varying glucose rate disturbance. Lyapunov function is used to check the stability analysis and input/output stability which play an important role in feedback control design for automatic control system. Numerical simulations are carried out to demonstrate our proposed result which show that the nonlinear fractional-order glucose-insulin systems are least stable in the existence of exogenous glucose infusion or meal disturbance. The concept of controllability and observability for the linearized control system of human glucose insulin system is used to have a feedback control for artificial pancreas.

==========

Ternary Non-stationary Subdivision Schemes in Hyperbolic Forms

Wardat us Salam
Department Mathematics, University of Education, Lahore

Non-stationary subdivision schemes become popular due to their ability to reproduce conic ssections. In this paper, ternary four point and five point non-stationary subdivision schemes are introduced that generates limiting curves of C1 and C4 continuity respectively. The capability of proposed four point interpolating subdivision scheme is that its limiting curve is more consistent to the control polyline than its trigonometric non-stationary subdivision scheme. The beauty of the proposed five point approximating subdivision scheme is that it reproduces parabolas and hyperbolas efficiently.

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Peristaltic flow of Jeffrey Six Constant Nanofluid Flow in an Endoscope with Non-uniform Vertical Tube

Aqila Shaheen
Department of Mathematics, CUI Lahore, Pakistan.

This paper analyze the peristaltic flow of Jeffrey six constant Nano fluid flow under the effect of endoscopic tube. The flow equations of non-Newtonian fluid for the two-dimensional tube in cylindrical coordinate system by using the assumption of low Reynold number and long wavelength approximation. The velocity equation is solved analytically by utilizing the homotopy perturbation technique, while the exact solutions are computed for temperature equation. The obtained expressions for velocity, concentration and temperature are sketched and the collision of assorted parameters are evaluating for transform peristaltic waves. The solution depends on thermophoresis number Nt, local nanoparticles Grashof number Gr, and Brownian motion number Nb. The obtained expressions for the velocity, temperature, and nanoparticles concentration profiles are plotted and the impact of various physical parameters are investigated for different peristaltic waves.

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Dynamics of Charged Viscous Dissipative Spherical Collapse with Perturbation Approach

Syed Munawar Shah
Department of Mathematics, The Islamia University of Bahawalpur, Pakistan.

This talk is devoted to the thermal evolution of shear-free charged gravitating compact objects which undergoes exhausted its inner heat during gravitational collapse. The evaluation equations have been formulated by applying the perturbation of first order to Einstein-Maxwell equations and Catteno-type heat transportation equation. We show that the temperature gradient (induced in the system owing to external effects), sufficiently depends upon multiple of relaxation time and total time during which the gravitating system oscillates. By taking some particular examples of nuclear matter, it has been noted that at a particular value of luminosity, the change in existing thermal profile is greater corresponding to higher values of times during which a system comes to rest position.

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A New Model of Quintessence Compact Stars in the Rastall Theory of Gravity

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In the present work, we study a new model of anisotropic compact stars in the regime of the Rastall theory. To solve the Rastall field equations we have used the Karori and Barua (KB) ansatz along with the quintessence dark energy characterized by a parameter ωq with $-1 < \omega q < -1$ 3. We present a comparative study to demonstrate the physical acceptance of our proposed model. We compare the numerical values of physical parameters obtained from our model with those of the general relativity (GR) model given by Bhar (Astrophys. Space Sci. 356, 309 (2015)) and observe that our model is more compatible (for some chosen values of Rastall dimensionless parameter $\gamma = \kappa\lambda$) with observational data than the GR model. For this analysis we have considered four different compact stars, SAXJ 1808 – 3658 (SSI), 4U 1820 − 30, VelaX −12 and PSRJ 1416 − 2230 with radii 7.07 km, 10 km, 9.99 km and 10.3 km, respectively. In this investigation we also present some physical aspects of the proposed model necessary to check the validity of the model and infer that our model is acceptable physically and geometrically.

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Effects of Thermal Fluctuations on Non-Minimal Regular Magnetic Black Hole

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We analyze the effects of thermal fluctuations on a regular black hole (RBH) of the non-minimal Einstein–Yang–Mill theory with gauge field of magnetic Wu–Yang type and a cosmological constant. We consider the logarithmic corrected entropy in order to analyze the thermal fluctuations corresponding to non-minimal RBH thermodynamics. In this scenario, we develop various important thermodynamical quantities, such as entropy, pressure, specific heats, Gibb's free energy and Helmholtz free energy. We investigate the first law of thermodynamics in the presence of logarithmic corrected entropy and non-minimal RBH. We also discuss the stability of this RBH using various frameworks such as the γ factor (the ratio of heat capacities), phase transition, grand canonical ensemble and canonical ensemble. It is observed that the non-minimal RBH becomes globally and locally more stable if we increase the value of the cosmological constant.

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Gravitational Perfect Fluid Collapse in Gauss-Bonnet Gravity

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The Einstein Gauss-Bonnet theory of gravity is the low energy limit of heterotic super-symmetric string theory. This paper deals gravitational collapse of perfect fluid in Einstein Gauss-Bonnet gravity by considering the Lemaitre -

Tolman - Bondi metric. For this purpose, the closed form of exact solution of equations of motion has been determined by using the conservation of stress-energy tensor and the condition of marginally bound shells. It has been investigated that the presence of Gauss-Bonnet coupling term $\alpha > o$ and pressure of the fluid modifies the structure and time formation of singularity. In this analysis singularity form earlier than horizon, so end state of the collapse is a naked singularity depending on the initial data. But this singularity is weak and time like that goes against the investigation of general relativity.

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Existence Results for Nonlinear Coupled Boundary Value Problems

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Problems with coupled boundary conditions (BCs) have been focus of many studies not only because of a theoretical interest but also they have rich applications in mathematical biology, chemical systems, engineering, and life sciences. Systems with coupled BCs are used in the study of Lotka-Volterra models, reaction diffusion phenomena, interaction problems, and Sturm-Liouville problems. In this talk, the application of fixed point theorems is studied to investigate the existence of solutions of certain nonlinear second-order coupled boundary value problems with dependence on the first order ordinary derivatives of the solution functions applying coupled lower and upper solutions approach. The existence results are constructed with the help of Schauder's fixed point theorem and Arzela-Ascoli theorem. The developed results are generalized in the sense that they cover the existence criteria of various systems of boundary value problems in a unified way, which were previously been treated on a case-by-case basis. The applicability of the developed theoretical results is ensured by considering some examples.

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Numerical Solution of the SIR Model with Vaccination, Treatment and Incidence Rate Using Galerkin Scheme

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A numerical study is performed to examine the effect of vaccination, treatment and incidence rate on general SIR model for epidemics which represents the direct transmission of infectious diseases. Vaccination rate represents that how much the susceptible individuals vaccinate to become a part of recovered individuals. Treatment rate represents how much treat the infected individuals to become a part of recovered individuals. In order to solve it numerically, continuous Galerkin-Petrov method in particular cGP(2)-method is implemented to determine the solutions of the model. For cGP(2)- method two unknowns are impact on each time interval which have to be calculated by solving 2x2 block system. This method is an accurate of order three in the whole time interval and shows convergence of order four in the discrete time points. The impact of different physical parameters on the population dynamics of susceptible, infected and recovered individuals are discussed in detail and illustrated the results graphically. Find the basic reproduction number and examined the stability in this study. Furthermore, for validity and reliability of the mentioned method the results are compared with the results obtained from classical RK4-method.

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Generators of Maximal Subgroups of Harada-Norton and some Linear Groups

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Group theory, the ultimate theory for symmetry, is a powerful tool that has a direct impact on research in robotics, computer vision, computer graphics and medical image analysis. Symmetry is very important in chemistry researches and group theory is the tool that is used to determine symmetry. Usually, it is not only the symmetry of molecule but also the symmetries of some local atoms, molecular orbitals, rotations and vibrations of bonds, etc. that are important. Harada-Norton group is an example of a sporadic simple group. There are 14 maximal subgroups of

Harada-Norton group. Generators (also known as words) of 11 maximal subgroups are already known. The aim of this note is to give generators of remaining 3 maximal subgroups which is an op en problem mentioned on A World-wide-web Atlas of Group Representations, http://brauer.maths.qmul.ac.uk/Atlas. In this report we compute the generators of $A_6 \times A_6.D_8$, $2^{3+2+6}.(3 \times L_3(2))$ and $3^4 : 2.(A_4 \times A_4).4$. Moreover we also compute the generators for the Maximal subgroups of some linear groups.