

WEIGHTED ESTIMATORS OF POPULATION MEAN USING TWO AUXILIARY VARIABLES UNDER TWO PHASE SAMPLING

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ABSTRACT

To estimate the population mean \bar{Y} of the variate y under study using two phase sampling procedure, a class of new estimators has been proposed when the population means of main auxiliary variable X and secondary auxiliary variable Z is available. The class of new estimators has less mean square of error than the mean squares of some previous class of estimators. The theoretical results obtained have been illustrated by taking some empirical study.

KEY WORDS

Two phase sampling; Mean-Squared Error; Bias; Auxiliary Variable; Simple random Sampling without Replacement.

1. INTRODUCTION

In sampling theory an unremitting concern is to estimate population mean by ratio, product and regression methods of estimation in the presence of multi-auxiliary variables using two phase sampling procedure. In this context many of the ratio, product and regression type estimators has been proposed from time to time for single and double sampling. Double sampling was first used by Neyman (1938) to obtain information on strata size in stratified sampling. Sukhatme (1962), Unnikrishan & Kunte (1995) made several efforts to provide a lot of applications of two phase sampling.

When regression line passes through the origin and correlation between study variable Y and auxiliary variable X is positive, the ratio method of estimation is regarded as the most practicable for estimating the population mean, however in the case of negative correlation, the product method of estimation is employed quite effectively. By considering the work of Searls (1964), Das and Tripathi, (1978), Sen (1978), Das and Tripathi, (1980), Sisodia and Dwivedi (1981), Panday and Dubey (1988), Upadhyaya and Singh (1999), Khoshnevisan et al. (2007) and Sing et al. (2007), proposed ratio type estimators for population mean by using different known information of certain parameters e.g. coefficient of correlation, coefficient of variation and kurtosis with known mean of the auxiliary variables to improve the efficiency of the estimators. The product estimator is developed by Robson (1957) and it is rediscovered by Murthy (1964). Rao and Mudholkar (1967) and Srivastava (1971) and others have made the extension of the ratio estimators to the case where multiple auxiliary variables are used to increase the precision. An improved estimator was proposed by Hanif et al. (2010) using multi auxiliary variables to estimate mean of population in multiphase sampling and the

estimator was compared with other estimators in which full information was available in double sampling. The proposed estimator was efficient than Abu-Dayyeh and other estimators.

1.1 Expectation of Errors in Two Phase Sampling Procedure

Let a population of N units say $U = \{U_1, U_2, \dots, U_N\}$. Let y and (x, z) be the variate of interest and auxiliary characteristics respectively related to y assume real non-negative j th value (y_j, x_j, z_j) $j = 1, 2, \dots, N$ with population means \bar{Y} , \bar{X} , and \bar{Z} respectively. A large first phase sample of size n_1 units is selected from N units from the population by Simple random sampling without replacement (SRSWOR) and the characteristics x and z say (x_1, z_1) are measured on it. A smaller second phase sample of size n_2 is selected from n_1 by SRSWOR and the characters y and z say (y_2, z_2) are measured on it.

For a simple random sampling without replacement (SRSWOR), we have some assumptions as following,

$$\begin{aligned}
 E(e_{\bar{y}_2}) &= E(e_{\bar{z}_2}) = E(e_{\bar{x}_1}) = E(e_{\bar{z}_1}) = 0 \\
 E(e_{\bar{y}_2})^2 &= \theta_2 \bar{Y}^2 C_y^2 & E(e_{\bar{z}_2})^2 &= \theta_2 \bar{Z}^2 C_z^2 \\
 E(e_{\bar{x}_1})^2 &= \theta_1 \bar{X}^2 C_x^2 & E(e_{\bar{z}_1})^2 &= \theta_1 \bar{Z}^2 C_z^2 \\
 E(e_{\bar{y}_2} e_{\bar{z}_2}) &= \theta_2 \bar{Y} \bar{Z} C_y C_z \rho_{yz} & E(e_{\bar{z}_2} e_{\bar{x}_1}) &= \theta_1 \bar{Z}^2 C_z^2 \\
 E(e_{\bar{x}_1} e_{\bar{z}_1}) &= \theta_1 \bar{X} \bar{Z} C_x C_z \rho_{zx} & E(e_{\bar{y}_2} e_{\bar{x}_1}) &= \theta_2 \bar{Y} \bar{X} C_y C_x \rho_{yx}
 \end{aligned} \tag{1.1.1}$$

2. SOME AVAILABLE ESTIMATORS

We have presented some of the well known estimators for population mean which use the information on auxiliary information. Mean squares of errors have also been given with each of the estimators.

Samiuddin and Hanif's (2006) Chain Ratio Estimator-I

$$t_1 = \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}} \tag{2.1}$$

$$MSE(t_1) = \bar{Y}^2 \left[\theta_1 \left(C_x^2 + 2C_x (C_y \rho_{xy} + C_z \rho_{xz}) \right) + \theta_2 \left(C_y^2 + C_z (C_z + 2C_y \rho_{yz}) \right) \right] \tag{2.2}$$

Samiuddin and Hanif's (2006) Chain Ratio Estimator-II

$$t_2 = \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2} \tag{2.3}$$

$$MSE(t_2) = \bar{Y}^2 \left[\theta_2 \left(C_y^2 + C_z^2 - 2C_y C_z \rho_{yz} \right) + \theta_1 \left(C_x^2 - 2C_y C_x \rho_{yx} + 2C_x C_z \rho_{xz} \right) \right] \tag{2.4}$$

Khaire and Srivastava (1981) Regression cum Ratio Estimator

$$t_3 = \left[\bar{y}_2 + b_{yx} (\bar{X} - \bar{x}_1) \right] \frac{\bar{Z}}{\bar{z}_2} \tag{2.5}$$

$$MSE(t_3) = \bar{Y}^2 \left[\theta_2 (C_y^2 + C_z^2 - 2C_y C_z \rho_{yz}) - \theta_1 C_y \rho_{xy} (C_y \rho_{xy} - 2C_z \rho_{xz}) \right] \tag{2.6}$$

Singh et al. (2005) Ratio cum Product Estimator

$$t_4 = \bar{y}_2 \left(\frac{\bar{X} + \rho_{xz}}{\bar{x}_1 + \rho_{xz}} \right) \left(\frac{\bar{z}_1 + \rho_{xz}}{\bar{Z} + \rho_{xz}} \right) \tag{2.7}$$

$$MSE(t_4) = \theta_1 \bar{Y}^2 \left[C_y^2 + C_x^2 \mu_1^2 + C_z^2 \mu_2^2 - 2\mu_1 C_y C_x \rho_{xy} - 2\mu_2 C_y C_z \rho_{yz} + 2\mu_1 \mu_2 C_z C_x \rho_{xz} \right] \tag{2.8}$$

$$\left. \begin{aligned} \mu_1 &= \frac{\bar{X}}{\bar{X} + \rho_{xz}} & \mu_2 &= \frac{\bar{Z}}{\bar{Z} + \rho_{xz}} \end{aligned} \right\}$$

3. 1st THE SUGGESTED ESTIMATOR

Motivated by Samiuddin and Hanif's (2006) chain ratio estimators, using x and known auxiliary variables, consider $t_1 = \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}}$ and $t_2 = \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2}$ as

$\hat{Y}_1 = \delta t_1 + (1 - \delta)t_2$ is a new proposed estimator with a constraint $0 < \delta < 1$ as

$$\hat{Y}_1 = \delta \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}} + (1 - \delta) \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2}; \quad 0 < \delta < 1 \tag{3.1}$$

where x_1 and z_1 are the sample means from sample at 1st and x_2, z_2 and y_2 are the means from sample at 2nd phase. δ and $1 - \delta$ are weights attached with t_1 and t_2 respectively.

To obtain Bias and MSE,s, consider

$$\bar{x}_1 = \bar{X} + e_{\bar{x}_1}; \quad \bar{y}_2 = \bar{Y} + e_{\bar{y}_2}; \quad \bar{z}_1 = \bar{Z} + e_{\bar{z}_1} \quad \bar{z}_2 = \bar{Z} + e_{\bar{z}_2}$$

Thus (3.1) is the expended form to obtain MSE,s to the first degree of approximation.

$$\hat{Y}_1 = \bar{Y} + e_{\bar{y}_2} + \delta \frac{\bar{Y}}{\bar{Z}} e_{\bar{z}_1} + (2\delta - 1) \frac{\bar{Y}}{\bar{X}} e_{\bar{x}_1} + (\delta - 1) \frac{\bar{Y}}{\bar{Z}} e_{\bar{z}_2} \tag{3.2}$$

Now MSE can defined as

$$MSE(\hat{Y}_1) = E(\hat{Y}_1 - \bar{Y})^2$$

$$= [e_{\bar{y}_2} + \delta \frac{\bar{Y}}{\bar{Z}} e_{\bar{z}_1} + (2\delta - 1) \frac{\bar{Y}}{\bar{X}} e_{\bar{x}_1} + (\delta - 1) \frac{\bar{Y}}{\bar{Z}} e_{\bar{z}_2}]^2;$$

$$= \bar{Y}^2 [\theta_2 C_y^2 + (2\delta^2 - \delta)\theta_1 C_z^2 + (2\delta - 1)^2 \theta_1 C_x^2 + (\delta - 1)^2 \theta_2 C_z^2 + \theta_1 \delta \rho_{yz} C_y C_z$$

$$+ (2\delta - 1)\theta_1 \rho_{xy} C_x C_y + (\delta - 1)\theta_2 \rho_{yz} C_y C_z + (2\delta - 1)^2 \theta_1 \rho_{yz} C_y C_z] \tag{3.3}$$

where

$$\theta_1 = \frac{1}{n_1} - \frac{1}{N}; \quad \theta_2 = \frac{1}{n_2} - \frac{1}{N} \quad \text{and}$$

$$\delta = \frac{\theta_1 A_1 + \theta_2 A_2}{\theta_1 A_3 + \theta_2 A_4}; \quad A_1 = C_z^2 + 4C_x^2 - 2\rho_{xy}C_xC_y + 3\rho_{yz}C_yC_z; \quad A_2 = 2C_z^2 - \rho_{yz}C_yC_z$$

$$A_3 = 4C_z^2 + 8C_x^2 + 8\rho_{yz}C_yC_z; \quad A_4 = 2C_z^2$$

Now Bias can be obtained by using 2nd degree approximation as

$$\text{Bias}(\hat{Y}_1) = E(\hat{Y}_1) - \bar{Y} = \bar{Y} \left[\theta_2 C_y^2 + (1-\delta)\theta_1 C_x^2 - \delta\theta_2 C_z^2 + \theta_1 \rho_{xz} C_x C_z \right. \\ \left. - \theta_2 \rho_{yz} C_y C_z + (2\delta-1)\theta_1 \rho_{xy} C_x C_y \right] \quad (3.4)$$

Remark 3.1

If $\delta \rightarrow 0$, the new suggested estimator \hat{Y}_1 will have very close resemblance of Samiuddin and Hanif (2006) estimator t_2 , i.e.

$$\hat{Y}_1 = \delta t_1 + (1-\delta)t_2 \rightarrow t_2 = \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2}$$

The empirical relation $\text{MSE}(\hat{Y}_1) < \text{MSE}(t_2)$ do holds good, so for the reason \hat{Y}_1 is preferred over t_2 .

Remark 3.2

If $\delta \rightarrow 1$, the new suggested estimator \hat{Y}_1 will tend to approximate Samiuddin and Hanif (2006) estimator t_1 , i.e.

$$\hat{Y}_1 = \delta t_1 + (1-\delta)t_2 \rightarrow t_1 = \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}}$$

The empirical relation $\text{MSE}(\hat{Y}_1) < \text{MSE}(t_1)$ do holds good, so for the reason \hat{Y}_1 is preferred over t_1 .

4. 2nd SUGGESTED ESTIMATOR

Using $t_1 = \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}}$ and $t_2 = \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2}$ for known auxiliary information, another new

estimator is proposed as $\hat{Y}_2 = \delta t_2 + (1-\delta)t_1$ with a constraint $0 < \delta < 1$. The new estimator can be written as

$$\hat{Y}_2 = \delta \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2} + (1-\delta) \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}}; \quad 0 < \delta < 1 \quad (4.1)$$

Thus (4.1) is the expended form to obtain MSE,s to the first degree of approximation.

$$\hat{Y}_2 = \bar{Y} + e_{\bar{y}_2} + (1-\delta)\frac{\bar{Y}}{\bar{Z}}e_{\bar{z}_1} + (1-2\delta)\frac{\bar{Y}}{\bar{X}}e_{\bar{x}_1} - \delta\frac{\bar{Y}}{\bar{Z}}e_{\bar{z}_2} \tag{4.2}$$

Now MSE can defined as

$$\begin{aligned} MSE(\hat{Y}_2) &= E(\hat{Y}_2 - \bar{Y})^2 \\ &= [e_{\bar{y}_2} + (1-\delta)\frac{\bar{Y}}{\bar{Z}}e_{\bar{z}_1} + (1-2\delta)\frac{\bar{Y}}{\bar{X}}e_{\bar{x}_1} - \delta\frac{\bar{Y}}{\bar{Z}}e_{\bar{z}_2}]^2 \\ &= \bar{Y}^2 \left[(1-3\delta+2\delta^2)\theta_1 C_z^2 + (1-2\delta)^2\theta_1 C_x^2 + \delta^2\theta_2 C_z^2 + \theta_2 C_y^2 + (2\delta-1)^2\theta_1 \rho_{xz} C_x C_z \right. \\ &\quad \left. + (1-\delta)\theta_1 \rho_{yz} C_y C_z + (1-2\delta)\theta_1 \rho_{xy} C_x C_y - \delta\theta_2 \rho_{yz} C_y C_z \right] \tag{4.3} \end{aligned}$$

$$\delta = \frac{\theta_1 B_1 + \theta_2 B_2}{\theta_1 B_3 + \theta_2 B_4};$$

where

$$\begin{aligned} B_1 &= C_z^2 + 4C_x^2 - 2\rho_{xy} C_x C_y + 3\rho_{yz} C_y C_z; & B_2 &= 2C_z^2 - \rho_{yz} C_y C_z \\ B_3 &= 4C_z^2 + 8C_x^2 + 8\rho_{yz} C_y C_z; & B_4 &= 2C_z^2 \end{aligned}$$

Now Bias can be obtained by using 2nd degree approximation as

$$\begin{aligned} Bias(\hat{Y}_2) &= E(\hat{Y}_2) - \bar{Y} = \bar{Y} \left[\delta(\theta_2 C_z^2 + \theta_1 C_x^2) + \theta_1 \rho_{xz} C_z C_x \right. \\ &\quad \left. + (\theta_1 - 2\theta_1 \delta) C_x C_y \rho_{xy} + (\theta_1 - \delta\theta_1 - \theta_2) C_y C_z \rho_{yz} \right] \tag{4.4} \end{aligned}$$

Remark 4.1

If $\delta \rightarrow 0$, the new suggested estimator \hat{Y}_2 will have very close resemblance of Samiuddin and Hanif (2006) estimator t_2 , i.e.

$$\hat{Y}_2 = \delta t_2 + (1-\delta)t_1 \quad \rightarrow \quad t_1 = \bar{y}_2 \frac{\bar{x}_1}{\bar{X}} \frac{\bar{z}_1}{\bar{Z}}$$

$MSE(\hat{Y}_2) < MSE(t_1)$ and the empirical relation to hold s good, so for the reason \hat{Y}_2 is preferred over t_1 .

Remark 4.2

If $\delta \rightarrow 1$, the new suggested estimator \hat{Y}_2 will tend to approximate Samiuddin and Hanif (2006) estimator t_2 , i.e.

$$\hat{Y}_2 = \delta t_2 + (1-\delta)t_1 \quad \rightarrow \quad t_2 = \bar{y}_2 \frac{\bar{X}}{\bar{x}_1} \frac{\bar{Z}}{\bar{z}_2}$$

$MSE(\hat{Y}_2) < MSE(t_2)$ and the empirical relation do holds good, so for the reason \hat{Y}_2 is preferred over t_2 .

6. EMPIRICAL STUDY

To comprise a forceful inspiration about the gain in efficiency of the proposed family over estimators in the literature, we take the four observed populations. The source of population, variable y , auxiliary variables X and Z , population size N , sample size at each phase (n_1, n_2) are given in table A-1 in appendix. The comparison is made with the estimators of population mean by Samiuddin and Hanif (2006), Khair and Srivastava (1981), and Sing et al. (2005). Mean squares of errors for these estimators and proposed estimators are given in table A-2 in appendix. The relative percent efficiency of the proposed estimators over other well known estimators is given in table-1 given below.

Table 1:
Percentage Relative Efficiencies of proposed estimators \hat{Y}_1 and \hat{Y}_2 with t_1, t_2, t_3 and t_4

Estimator	Population-I		Population-II	
	\hat{Y}_1	\hat{Y}_2	\hat{Y}_1	\hat{Y}_2
t_1	784.602	819.1344	775.0546	761.1861
t_2	442.992	462.4889	320.9655	315.2222
t_3	410.005	428.0502	370.8699	364.2337
t_4	442.081	461.5381	319.3601	313.6455
Estimator	Population-III		Population-IV	
	\hat{Y}_1	\hat{Y}_2	\hat{Y}_1	\hat{Y}_2
t_1	756.265	781.9358	633.808	630.3374
t_2	398.4089	411.9326	349.0461	347.1348
t_3	398.813	412.3512	313.5134	311.7967
t_4	397.3514	410.8391	347.6414	345.7378

7. CONCLUSION

The results of comparative study in the tables 1 are clearly indicating that the suggested estimators are more efficient than Samiuddin and Hanif (2006), Khair and Srivastava (1981), and Sing et al. (2005).

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APPENDIX-A

Table A-1
Description of Populations

S#	Population	Study Variable (Y)	Main Auxiliary Variable (X)	Secondary Auxiliary Variable (Z)	(N, n ₁ , n ₂)
1	Population Census Reports of Okara District (1998) Pakistan	Population Matric and above	Primary and Below	Population both Sex	(300, 60, 12)
2	Advanced Applied Statistical Linear Model (Neter and Kutner, 2004)	Total Serious Crimes	No of unemployed	Total Population	(440, 88, 18)
3	Advanced Applied Linear Model (Nachstshemim, Neter and Kutner, 2004)	Total Cost of Claims	Total no of Interventions	No of Emergency Room Visits	(778, 156, 31)
4	Advanced Applied Linear Model (Nachstshemim, Neter and Kutner, 2004)	Sale Price of Residence	Finished Area of Residence (Square feet)	No of Bed Rooms in the Residence	(552, 104, 21)

Table A-2
Mean Square Errors (MSE) of Estimators

Population	ESTIMATOR					
	\hat{Y}_1	\hat{Y}_2	t_1	t_2	t_3	t_4
1	31.46437381	30.13794687	246.8702791	139.3846517	129.0055466	139.0981172
2	17.12605656	17.43808743	132.7362899	54.96872815	63.51538271	54.69378449
3	23.4364804	22.66706792	177.2419272	93.37303344	93.46793011	93.12517604
4	9.727675165	9.781235049	61.65478076	33.95407282	30.49756975	33.81743084